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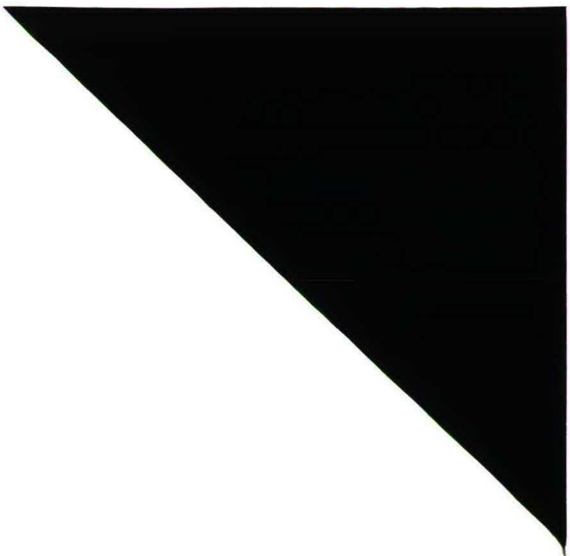

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Abstract

There exists an extensive literature about economies with price rigidities, where some constraints on the set of admissible price systems are exogenously given. In this paper two models of a political economic system are described where the price rigidities are endogenously chosen by political candidates. Sufficient conditions for the existence of a political economic equilibrium are given for both models. Moreover, it is shown that, generically, political candidates will not choose price regulations leading to a Walrasian equilibrium, but instead will impose price regulations upon the economic system which exclude all Walrasian equilibria and therefore lead to rationing of consumers.

1 Introduction

In Herings (1994) a model of the political economic system has been given such that the price regulations imposed on the economic system resulted endogenously. Moreover, in that paper an example has been given where the price regulations are chosen in such a way by the political candidates that the Walrasian equilibrium price system is excluded. However, it is not clear whether this is the typical case. Moreover, it is clear that it is possible to construct examples where both political candidates propose price regulations such that a Walrasian equilibrium results.

In this paper different assumptions are made with respect to the economy, guaranteeing that the indirect utility functions of the consumers satisfy certain differentiability properties. Then it will be possible to answer the question whether, generically, political candidates choose price regulations excluding a Walrasian equilibrium. Moreover, under these assumptions it is possible to formulate another appealing model of the political economic system where political candidates are considered to choose only among local options given some status quo. More precisely, political candidates have the possibility to choose directions of motion away from the status quo and the possibility to stay at the status quo. Acquiring information concerning proposals very far away from the status quo, like voting behaviour at such a proposal, is often very expensive. Moreover, institutional restrictions or commitments made in the past may rule out large changes. This provides some motivation for the restriction to local options. The pay-offs for the political candidates are determined by the marginal change in the number of votes corresponding to a certain direction of motion. An equilibrium of the resulting game is called a directional political economic equilibrium. This way of modelling the political system is inspired by Coughlin and Nitzan (1981).

In Section 2 the assumptions made with respect to the political economic system are described and the model of Herings (1994) is presented. In Section 3 some known results with respect to the partial derivatives of the indirect utility functions of the consumers are given. Moreover, the model of a political economic system where political candidates choose between local options given some status quo is presented. The status quo will be assumed to be some Walrasian equilibrium of the economy. It is shown that a directional political economic equilibrium in pure strategies exists and a characterization of the equilibrium actions of the political candidates is given. In Section 4 it is shown that, generically, both in the model of Herings (1994) and in the model of Section 3, a Walrasian equilibrium is unstable in a political economic system, unless there is only one consumer in the economy, or there is only one commodity. For the model of Herings (1994) it is shown that, generically, given a proposal of a political candidate corresponding to a Walrasian equilibrium, there exist price regulations excluding this Walrasian equilibrium and being better responses than proposing this Walrasian equilibrium. For the model of Section 3 it is shown that, generically, both political candidates choose to move away from the status quo, being any Walrasian equilibrium. In Section 5 the directional political economic equilibrium is determined for the same example as given in Herings (1994). It will be shown that in this example the unique Walrasian equilibrium is indeed unstable.

2 The Political Economic System

In the following, for $k \in \mathbb{N}$, define $I_k = \{1, \ldots, k\}$, $Q^k = \{q \in \mathbb{R}^k \mid 0 \leq q_j \leq 1, \ \forall j \in I_k\}$, let 0^k be a k-dimensional vector of zeroes, let 1^k be a k-dimensional vector of ones, let $-\infty^k$ be a k-dimensional vector with every component equal to $-\infty$, and let $+\infty^k$ be a k-dimensional vector with every component equal to $+\infty$. The set of extended real numbers is denoted by $\mathbb{R}^*$ and the k-dimensional Cartesian product of this set by $\mathbb{R}^{*k}$. If $x^1, x^2 \in \mathbb{R}^{*k}$, then $x^1 \leq x^2$ means $x_j^1 \leq x_j^2$, $\forall j \in I_k$, $x^1 < x^2$ means $x^1 \leq x^2$ and there exists $j \in I_k$ such that $x_j^1 < x_j^2$, and $x^1 \ll x^2$ means $x_j^1 < x_j^2$, $\forall j \in I_k$. Similarly, $\geq, >$, and $\gg$ are defined. The set $\{x \in \mathbb{R}^{*k} \mid x \geq 0^k\}$ is denoted by $\mathbb{R}_+^{*k}$ and the set $\{x \in \mathbb{R}^{*k} \mid x \gg 0^k\}$ is denoted by $\mathbb{R}_{++}^{*k}$. The sets $\mathbb{R}_+^k$ and $\mathbb{R}_{++}^k$ are defined similarly.

It is assumed in this paper that there are $M \in \mathbb{N}$ consumers, indexed by $i \in I_M$, $N \in \mathbb{N} \setminus \{1\}$ commodities, indexed by $j \in I_N$, and two political candidates, indexed by $k \in I_2$. Every consumer $i \in I_M$ is characterized by the consumption set X^i, the utility function $u^i : X^i \to \mathbb{R}$, representing the preference relation $\preceq^i$, and the initial endowment ω^i. Notice that there is no loss of generality in assuming that $u^i(X^i) \subset (0,1)$. Together this constitutes the economy $\mathcal{E} = (X^i, \preceq^i, \omega^i)_{i \in I_M}$.

The rationing function, specifying the admissible rationing schemes, is given by the pair $(\tilde{l}, \tilde{L})$ with $\tilde{l} : Q^N \to -\mathbb{R}_+^{MN}$ the rationing function on supply and $\tilde{L} : Q^N \to \mathbb{R}_+^{MN}$ the rationing function on demand. The set $\prod_{i \in I_M} X^i$ is denoted by X. If $x = (x^1, \ldots, x^M)$ is an element of X, then $x_j = (x_j^1, \ldots, x_j^M)^\top$, $\forall j \in I_N$. Moreover, $\omega = (\omega^1, \ldots, \omega^M)$ and $\omega_j = (\omega_j^1, \ldots, \omega_j^M)^\top$, $\forall j \in I_N$. For every $i \in I_M$, for every $j \in I_N$, component $(i-1)N + j$ of $\tilde{l}$ is denoted by $\tilde{l}_j^i$. Moreover, $\tilde{l}^i = (\tilde{l}_1^i, \ldots, \tilde{l}_N^i)^\top$, $\forall i \in I_M$, and $\tilde{l}_j = (\tilde{l}_j^1, \ldots, \tilde{l}_j^M)^\top$, $\forall j \in I_N$. The same notation is used for the function $\tilde{L}$, for a rationing scheme on supply $l \in -\mathbb{R}_+^{*MN}$, and for a rationing scheme on demand $L \in \mathbb{R}_+^{*MN}$.

Given a price system $p \in \mathbb{R}^N$ and a rationing scheme $(l^i, L^i) \in -\mathbb{R}_+^{*N} \times \mathbb{R}_+^{*N}$ of consumer $i \in I_M$, the budget set $\beta^i(p, l^i, L^i)$ of consumer i is defined by

$$\beta^i(p, l^i, L^i) = \{x^i \in X^i \mid p \cdot x^i \leq p \cdot \omega^i \text{ and } l^i \leq x^i - \omega^i \leq L^i\}$$

and the set $\delta^i(p, l^i, L^i)$ denotes the set of best elements of $\beta^i(p, l^i, L^i)$ for $\preceq^i$. The set $\overline{P}$ of price regulations is defined by

$$\overline{P} = \{(\underline{p}, \overline{p}) \in \mathbb{R}^N_+ \times \mathbb{R}^N_+ \mid \underline{p} \leq \overline{p}, \underline{p}_N = \overline{p}_N = 1\}.$$

An element $(\underline{p}, \overline{p}) \in \overline{P}$ induces the set of admissible price systems $P_{(\underline{p},\overline{p})}$, defined by

$$P_{(\underline{p},\overline{p})} = \{p \in \mathbb{R}^N_+ \mid \underline{p} \leq p \leq \overline{p}\}.$$

So commodity N is assumed to be a numeraire commodity with price equal to 1. Given $(X^i, \preceq^i, \omega^i)_{i \in I_M}$ and $(\tilde{l}, \tilde{L})$, $(\underline{p}, \overline{p})$ yields the economy

$$\tilde{\mathcal{E}}(\underline{p}, \overline{p}) = ((X^i, \preceq^i, \omega^i)_{i \in I_M}, P_{(\underline{p},\overline{p})}, (\tilde{l}, \tilde{L})).$$

A Drèze equilibrium is defined as follows.

Definition 2.1 (Drèze Equilibrium)
Let some $(\underline{p}, \overline{p}) \in \overline{P}$ be given. A Drèze equilibrium of the economy $\tilde{\mathcal{E}}(\underline{p}, \overline{p}) = ((X^i, \preceq^i, \omega^i)_{i \in I_M}, P_{(\underline{p},\overline{p})}, (\tilde{l}, \tilde{L}))$ is an element

$$(p^*, l^*, L^*, x^*) \in P_{(\underline{p},\overline{p})} \times \tilde{l}(Q^N) \times \tilde{L}(Q^N) \times X$$

satisfying

1. *for every consumer $i \in I_M$, $x^{*i} \in \delta^i(p^*, l^{*i}, L^{*i})$,*
2. $\sum_{i \in I_M} x^{*i} - \sum_{i \in I_M} \omega^i = 0^N$,
3. *for every commodity $j \in I_{N-1}$, $x^{*i'}_j - \omega^{i'}_j = l^{*i'}_j$ for some consumer $i' \in I_M$ implies $x^{*i}_j - \omega^i_j < L^{*i}_j$, $\forall i \in I_M$, and $x^{*i'}_j - \omega^{i'}_j = L^{*i'}_j$ for some consumer $i' \in I_M$ implies $x^{*i}_j - \omega^i_j > l^{*i}_j$, $\forall i \in I_M$,*
4. *for every commodity $j \in I_{N-1}$, $p^*_j > \underline{p}_j$ implies $l^{*i}_j < x^{*i}_j - \omega^i_j$, $\forall i \in I_M$, and $p^*_j < \overline{p}_j$ implies $L^{*i}_j > x^{*i}_j - \omega^i_j$, $\forall i \in I_M$,*
5. $l^{*i}_N < x^{*i}_N - \omega^i_N < L^{*i}_N$, $\forall i \in I_M$.

The set of Drèze equilibria of the economy $\tilde{\mathcal{E}}(\underline{p}, \overline{p})$ is denoted by $\tilde{E}^D(\underline{p}, \overline{p})$.

The function $\hat{p} : Q^{N-1} \times \overline{P} \to \mathbb{R}^N_+$ and the functions $\hat{l} : Q^{N-1} \to -\mathbb{R}^{MN}_+$ and $\hat{L} : Q^{N-1} \to \mathbb{R}^{MN}_+$ are obtained by defining, for every $j \in I_{N-1}$,

$$\begin{aligned} \hat{p}_j(q, \underline{p}, \overline{p}) &= \max\left(\left\{\underline{p}_j, \min(\{\underline{p}_j(2 - 3q_j) + \overline{p}_j(3q_j - 1), \overline{p}_j\})\right\}\right), && \forall (q, \underline{p}, \overline{p}) \in Q^{N-1} \times \overline{P}, \\ \hat{l}_j(q) &= \tilde{l}_j\left(\inf(\{1^N, (3q^\top, 1)^\top\})\right), && \forall q \in Q^{N-1}, \\ \hat{L}_j(q) &= \tilde{L}_j\left(\inf(\{1^N, 31^N - (3q^\top, 2)^\top\})\right), && \forall q \in Q^{N-1}, \end{aligned}$$

and by defining

$$
\begin{aligned}
\widehat{p}_N(q,\underline{p},\overline{p}) &= 1, && \forall (q,\underline{p},\overline{p}) \in Q^{N-1} \times \underline{\overline{P}}, \\
\widehat{l}_N(q) &= \widetilde{l}_N \left(\inf(\{1^N, (3q^\top, 1)^\top\}) \right), && \forall q \in Q^{N-1}, \\
\widehat{L}_N(q) &= \widetilde{L}_N \left(\inf(\{1^N, 31^N - (3q^\top, 2)^\top\}) \right), && \forall q \in Q^{N-1}.
\end{aligned}
$$

For every consumer $i \in I_M$, the reduced demand relation $\widehat{\delta}^i : Q^{N-1} \times \underline{\overline{P}} \to \mathbb{R}^N$ is defined by

$$\widehat{\delta}^i(q,\underline{p},\overline{p}) = \delta^i(\widehat{p}(q,\underline{p},\overline{p}), \widehat{l}^i(q), \widehat{L}^i(q)), \ \forall (q,\underline{p},\overline{p}) \in Q^{N-1} \times \underline{\overline{P}}.$$

The notational conventions used for $\widetilde{l}$ and $\widetilde{L}$ are also used for $\widehat{l}$ and $\widehat{L}$. Let some $(\underline{p},\overline{p}) \in \underline{\overline{P}}$ be given. If, for some $q^* \in Q^{N-1}$, there exists $x^{*i} \in \widehat{\delta}^i(q^*,\underline{p},\overline{p})$, $\forall i \in I_M$, such that $\sum_{i \in I_M} x^{*i} = \sum_{i \in I_M} \omega^i$, then, under weak assumptions with respect to consumption sets and preference relations, $(\widehat{p}(q^*,\underline{p},\overline{p}), \widehat{l}(q^*), \widehat{L}(q^*), x^*) \in \widetilde{E}^D(\underline{p},\overline{p})$, called a Drèze equilibrium of $\widetilde{\mathcal{E}}(\underline{p},\overline{p})$ induced by q^*. The set $\widetilde{Q}^D(\underline{p},\overline{p})$ is defined by $\widetilde{Q}^D(\underline{p},\overline{p}) = \{q^* \in Q^{N-1} \mid \sum_{i \in I_M} \omega^i \in \sum_{i \in I_M} \widehat{\delta}^i(q^*,\underline{p},\overline{p})\}$. Moreover, under weak assumptions with respect to the economy, no Drèze equilibria are lost by restricting attention to the Drèze equilibria induced by some element of $\widetilde{Q}^D(\underline{p},\overline{p})$.

Every political candidate $k \in I_2$ has a set of admissible price regulations $A^k \subset \underline{\overline{P}}$, determining the set of admissible actions $\mathcal{A}^k$, defined by $\mathcal{A}^k = \{(a^k, q^k) \in A^k \times Q^{N-1} \mid q^k \in \widetilde{Q}^D(a^k)\}$. The set of admissible actions corresponding to the set of price regulations $\underline{\overline{P}}$ is denoted by $\mathcal{A}$. The indirect utility function $\widetilde{v}^i : \mathcal{A} \to [0,1]$ of consumer $i \in I_M$ is defined by associating with every $(a,q) \in \mathcal{A}$ the real number $\widetilde{v}^i(a,q)$ satisfying $\widetilde{v}^i(a,q) = u^i(x^{*i})$, $\forall x^{*i} \in \widehat{\delta}^i(q,a)$. For every $i \in I_M$, for every $k \in I_2$, the voting function $\pi^{ik} : (0,1) \times (0,1) \to [0,1]$ describes the expectations of a political candidate about the voting behaviour of consumer i concerning political candidate k, i.e., $\pi^{ik}(v^1, v^2)$ is the probability a political candidate assigns to the event that consumer $i \in I_M$ votes for political candidate k if the proposal of political candidate 1 yields consumer i a utility level v^1 and the proposal of political candidate 2 yields consumer i a utility level v^2. For the sake of simplicity it is assumed that both political candidates have the same expectations and that there are no abstentions, so, for every $i \in I_M$, $\pi^{i1}(\widetilde{v}^i(a^1,q^1), \widetilde{v}^i(a^2,q^2)) + \pi^{i2}(\widetilde{v}^i(a^1,q^1), \widetilde{v}^i(a^2,q^2)) = 1$, $\forall ((a^1,q^1),(a^2,q^2)) \in \mathcal{A}^1 \times \mathcal{A}^2$.

It is assumed that political candidates maximize their expected plurality in the elections. Therefore, the pay-off function $K^1 : \mathcal{A}^1 \times \mathcal{A}^2 \to \mathbb{R}$ of political candidate 1 is defined by

$$
\begin{aligned}
&K^1\left((a^1,q^1),(a^2,q^2)\right) \\
&\quad = \sum_{i \in I_M} \pi^{i1}\left(\widetilde{v}^i(a^1,q^1), \widetilde{v}^i(a^2,q^2)\right) \\
&\qquad - \sum_{i \in I_M} \pi^{i2}\left(\widetilde{v}^i(a^1,q^1), \widetilde{v}^i(a^2,q^2)\right), \ \forall \left((a^1,q^1),(a^2,q^2)\right) \in \mathcal{A}^1 \times \mathcal{A}^2.
\end{aligned}
$$

The pay-off function $K^2 : \mathcal{A}^1 \times \mathcal{A}^2 \to \mathbb{R}$ of political candidate 2 is easily seen to be given by $K^2 = -K^1$. A political economic equilibrium of the political economic system $\hat{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_M}, (\tilde{l}, \tilde{L}), (A^k, (\pi^{ik})_{i \in I_M})_{k \in I_2})$ is defined as follows.

Definition 2.2 (Political Economic Equilibrium)
A political economic equilibrium of the political economic system $\hat{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_M}, (\tilde{l}, \tilde{L}), (A^k, (\pi^{ik})_{i \in I_M})_{k \in I_2})$ *is a Nash equilibrium of the mixed extension of the game* $\mathcal{G} = (\mathcal{A}^1, \mathcal{A}^2, K^1, K^2)$.

For the main results of this paper the following assumptions are made.

A1. For every consumer $i \in I_M$, the consumption set X^i is equal to $\mathbb{R}^N_{++}$.

A2. For every consumer $i \in I_M$, the utility function $u^i : X^i \to (0,1)$ is twice continuously differentiable, $u^i(X^i) = (0,1)$, has no critical point, and represents the preference relation $\preceq^i$ being complete, transitive, continuous, strongly monotonic, strongly convex, of the class C^2, satisfying the boundary condition, and having non-zero Gaussian curvature.

A3. For every consumer $i \in I_M$, the initial endowment ω^i belongs to X^i.

A4. The rationing function $(\tilde{l}, \tilde{L})$ is flexible, market independent, continuously differentiable, monotonic, and, for every $j \in I_N$, $\partial_{q_j} \tilde{l}_j(\overline{q}) \neq 0^M$, $\forall \overline{q} \in Q^N$, and $\partial_{q_j} \tilde{L}_j(\overline{q}) \neq 0^M$, $\forall \overline{q} \in Q^N$.

A5. For every political candidate $k \in I_2$, $A^k = \{(\underline{p}, \overline{p}) \in \overline{P} \mid \underline{p} = \overline{p}\}$.

A6. For every consumer $i \in I_M$, for every political candidate $k \in I_2$, the voting function $\pi^{ik} : (0,1) \times (0,1) \to [0,1]$ is continuously differentiable.

For some results it is sufficient to take Assumption A6, but for other results Assumption A7 is needed.

A7. For every consumer $i \in I_M$, the voting function $\pi^{i1} : (0,1) \times (0,1) \to [0,1]$ is twice continuously differentiable, $\partial_{v^1} \pi^{i1}(\overline{v}^1, \overline{v}^2) > 0$, $\forall (\overline{v}^1, \overline{v}^2) \in (0,1) \times (0,1)$, and $\partial_{v^2} \pi^{i1}(\overline{v}^1, \overline{v}^2) < 0$, $\forall (\overline{v}^1, \overline{v}^2) \in (0,1) \times (0,1)$. For every consumer $i \in I_M$, the voting function $\pi^{i2} : (0,1) \times (0,1) \to [0,1]$ is twice continuously differentiable, $\partial_{v^1} \pi^{i2}(\overline{v}^1, \overline{v}^2) < 0$, $\forall (\overline{v}^1, \overline{v}^2) \in (0,1) \times (0,1)$, and $\partial_{v^2} \pi^{i2}(\overline{v}^1, \overline{v}^2) > 0$, $\forall (\overline{v}^1, \overline{v}^2) \in (0,1) \times (0,1)$.

Notice that, for every consumer $i \in I_M$, a preference relation $\preceq^i$ satisfying the conditions given in Assumption A2 can indeed be represented by a twice continuously differentiable utility function having no critical point. Moreover, there is no loss of generality involved in assuming that $u^i(X^i) = (0,1)$, since due to the Assumptions A1 and A2, there is no worst and no best element of X^i for $\preceq^i$. Assumption A7 guarantees that voters react in a natural way to decreases or increases in proposed utility levels. Notice that Assumption A7 implies that, for every $i \in I_M$, for every $k \in I_2$, $0 < \pi^{ik}(v^1, v^2) < 1$, $\forall (v^1, v^2) \in (0,1) \times (0,1)$.

Although the assumptions made in Herings (1994) in order to show the existence of a political economic equilibrium of the political economic system $\widehat{\mathcal{E}}$ are not implied by the Assumptions A1-A6, the proof of the following result is similar as the proof given in Herings (1994).

Theorem 2.3
Let the political economic system $\widehat{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_M}), (\widetilde{l}, \widetilde{L}), (A^k, (\pi^{ik})_{i \in I_M})_{k \in I_2})$ *satisfy the Assumptions A1-A6. Then there exists a political economic equilibrium of the political economic system* $\widehat{\mathcal{E}}$.
See Herings (1994), Theorem 4.6, page 21.

3 The Existence of a Directional Political Economic Equilibrium

First a few concepts introduced in Laroque (1978) have to be discussed. These concepts are needed to study some properties of the Drèze equilibria of the economy $\widetilde{\mathcal{E}}(\underline{p}, \overline{p})$ and of the indirect utility functions of the consumers at price regulations $(\underline{p}, \overline{p})$ with $\underline{p} = \overline{p}$ being close to p^*, where (p^*, x^*) with $p^*_N = 1$ is a Walrasian equilibrium of the economy $\mathcal{E}$.

Let $(X^i, u^i, \omega^i)_{i \in I_M}, (\widetilde{l}, \widetilde{L})$ satisfy the Assumptions A1-A4 and let a Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ be given, where $((X^i, u^i, \omega^i)_{i \in I_M})$ is assumed to denote the economy $((X^i, \preceq^i, \omega^i)_{i \in I_M})$ with, for every $i \in I_M$, $\preceq^i$ the preference relation being represented by u^i. Define the elements $\underline{q}(x^*) \in Q^{N-1}$ and $\overline{q}(x^*) \in Q^{N-1}$ by

$$\begin{aligned} \underline{q}(x^*) &= \inf\left(\left\{q \in Q^{N-1} \,\middle|\, \widehat{l}(q) \leq x^* - \omega\right\}\right), \\ \overline{q}(x^*) &= \sup\left(\left\{q \in Q^{N-1} \,\middle|\, \widehat{L}(q) \geq x^* - \omega\right\}\right). \end{aligned}$$

For every commodity $j \in I_{N-1}$, define the sets $\underline{I}_j(x^*)$ and $\overline{I}_j(x^*)$ by

$$\begin{aligned} \underline{I}_j(x^*) &= \left\{i \in I_M \,\middle|\, \widehat{l}^i_j(\underline{q}(x^*)) = x^{*i}_j - \omega^i_j\right\}, \\ \overline{I}_j(x^*) &= \left\{i \in I_M \,\middle|\, \widehat{L}^i_j(\overline{q}(x^*)) = x^{*i}_j - \omega^i_j\right\}, \end{aligned}$$

so these sets contain the consumers with in some sense minimal and maximal excess demand on the market of commodity j, respectively. Notice that $\widehat{l}_N(q) < x^*_N - \omega_N < \widehat{L}_N(q)$, $\forall q \in Q^{N-1}$.

Let $(X^i, u^i, \omega^i)_{i \in I_M}, (\tilde{l}, \tilde{L})$ satisfy the Assumptions A1-A4 and let a Walrasian equilibrium (p^*, x^*) with $p_N^* = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ be given. Let a sign vector $s \in \mathbb{S}^{N-1}$ with $I^0(s) = \emptyset$ be given. For every consumer $i \in I_M$, for every $(p_1, \dots, p_{N-1})^\top \in \mathbb{R}^{N-1}$, for every $a \in \mathbb{R}^{N-1}$, define the set $\beta^{i,s}((p_1, \dots, p_{N-1})^\top, a)$ by

$$\begin{aligned}\beta^{i,s}\left((p_1,\dots,p_{N-1})^\top, a\right) = \Big\{x^i \in X^i \Big| &(p_1,\dots,p_{N-1},1)^\top \cdot x^i = (p_1,\dots,p_{N-1},1)^\top \cdot \omega^i,\\ &x_j^i - \omega_j^i = \hat{l}_j^i(\underline{q}(x^*)) + a_j \text{ if } i \in \underline{I}_j(x^*) \text{ and } j \in I^-(s),\\ &x_j^i - \omega_j^i = \hat{L}_j^i(\overline{q}(x^*)) + a_j \text{ if } i \in \overline{I}_j(x^*) \text{ and } j \in I^+(s)\Big\},\end{aligned}$$

and define the set $\delta^{i,s}((p_1, \dots, p_{N-1})^\top, a)$ by

$$\delta^{i,s}\left((p_1,\dots,p_{N-1})^\top, a\right) = \left\{\overline{x}^i \in \beta^{i,s}((p_1,\dots,p_{N-1})^\top, a) \middle| \overline{x}^i \succeq^i x^i, \forall x^i \in \beta^{i,s}((p_1,\dots,p_{N-1})^\top, a)\right\}.$$

For every $(p_1, \dots, p_{N-1})^\top \in \mathbb{R}^{N-1}$, for every $a \in \mathbb{R}^{N-1}$, define the set $\zeta^s((p_1, \dots, p_{N-1})^\top, a)$ by

$$\begin{aligned}&\zeta^s\left((p_1,\dots,p_{N-1})^\top, a\right)\\ &\quad = \left\{\overline{z} \in \mathbb{R}^{N-1} \middle| \exists \overline{z}_N \in \mathbb{R}, (\overline{z}^\top, \overline{z}_N)^\top \in \textstyle\sum_{i \in I_M}\left(\delta^{i,s}((p_1,\dots,p_{N-1})^\top, a) - \{\omega^i\}\right)\right\}.\end{aligned}$$

In this way a relation $\zeta^s : \mathbb{R}^{N-1} \times \mathbb{R}^{N-1} \to \mathbb{R}^{N-1}$ is obtained. In Laroque (1978), Proposition 5.1, page 1134, it is shown that there exists a set O, being open in $\mathbb{R}^{N-1} \times \mathbb{R}^{N-1}$ and containing $((p_1^*, \dots, p_{N-1}^*)^\top, 0^{N-1})$, such that $\zeta^s_{|O}$ is a continuously differentiable function, denoted by z^s, so $z^s : O \to \mathbb{R}^{N-1}$. Therefore, the matrix of partial derivatives, $\partial_a z^s((p_1^*, \dots, p_{N-1}^*)^\top, 0^{N-1})$ is well-defined. The following definition is given in Laroque (1978), Definition 5.1, page 1135.

Definition 3.1 (Regular Walrasian equilibrium)
A Walrasian equilibrium (p^*, x^*) with $p_N^* = 1$ of the economy $\mathcal{E} = ((X^i, \preceq^i, \omega^i)_{i \in I_M})$ is regular if for every sign vector $s \in \mathbb{S}^{N-1}$ with $I^0(s) = \emptyset$ the matrix of partial derivatives $\partial_a z^s((p_1^*, \dots, p_{N-1}^*)^\top, 0^{N-1})$ is invertible.

The following result shows the importance of regular Walrasian equilibria. Notice that a Walrasian equilibrium (p^*, x^*) with $p_N^* = 1$ of the economy $\mathcal{E}$ is said to be locally unique if there exists a set being open in $\mathbb{R}^N \times \mathbb{R}^{MN}$ and containing (p^*, x^*), but not containing any other Walrasian equilibrium $(\overline{p}^*, \overline{x}^*)$ with $\overline{p}_N^* = 1$ of $\mathcal{E}$.

Theorem 3.2
Let the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M}, (\tilde{l}, \tilde{L}))$ satisfy the Assumptions A1-A4, and let (p^*, x^*) with $p_N^* = 1$ be a locally unique, regular Walrasian equilibrium of the economy $\mathcal{E}$ such that, for every $j \in I_{N-1}$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$. Then for every $(p, p) \in \overline{\underline{P}}$ there

exists $q(p) \in \tilde{Q}^D(p,p)$ *such that, for every* $i \in I_M$, *the function* $\widehat{v}^i : \mathbb{R}_+^{N-1} \to (0,1)$, *defined by associating with every* $(p_1,\dots,p_{N-1})^\top \in \mathbb{R}_+^{N-1}$ *the element*

$$\widehat{v}^i(p_1,\dots,p_{N-1}) = \tilde{v}^i\left((p_1,\dots,p_{N-1},1)^\top,(p_1,\dots,p_{N-1},1)^\top,q((p_1,\dots,p_{N-1},1)^\top)\right),$$

satisfies that $\widehat{v}^i(p_1^*,\dots,p_{N-1}^*) = u^i(x^{*i})$ *and* $\partial\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)$ *exists. Moreover, for every* $i \in I_M$,

$$\partial_{p_j}\widehat{v}^i(p_1^*,\dots,p_{N-1}^*) = -\partial_{x_N}u^i(x^{*i})(x_j^{*i} - \omega_j^i),\ \forall j \in I_{N-1}.$$

See Laroque (1978), Proposition 7.1, page 1144.

It follows from the results shown by Laroque that for every price system $p \in \overline{P}$ being close to p^*, there exists a uniquely determined Drèze equilibrium of the economy $\tilde{\mathcal{E}}(p,p)$ being close to (p^*,x^*).

The following three results show that the conditions given in Theorem 3.2 are satisfied for a typical economy. The set of all utility functions of a consumer $i \in I_M$ satisfying Assumption A2 is denoted by U^i. This set is given the topology induced by the C^2-topology. Define the set U by $U = \prod_{i\in I_M} U^i$ and give this set the product topology. Notice that, for every $u \in U$, $u = (u^1,\dots,u^M)$ with u^i denoting the utility function of consumer $i \in I_M$. The set of initial endowments satisfying Assumption A3 is denoted by Ω. The following result shows that, generically, an economy is regular.

Theorem 3.3

Let $(X^i)_{i\in I_M}$ *satisfy the Assumption A1. Then there exists an open and dense set* $\mathcal{U}^1$ *in* $U \times \Omega$ *such that, for every* $(u,\omega) \in \mathcal{U}^1$, *every Walrasian equilibrium* (p^*,x^*) *with* $p_N^* = 1$ *of the economy* $\mathcal{E} = ((X^i,u^i,\omega^i)_{i\in I_M})$ *is regular.*

See Wiesmeth (1979), Theorem, page 25.

The following result shows that a typical economy has a finite number of Walrasian equilibria (p^*,x^*) with $p_N^* = 1$, so every Walrasian equilibrium of a typical economy is locally unique. The finiteness of the number of Walrasian equilibria of a typical economy was first shown in Debreu (1970). Debreu (1970) gives a result with the demand functions of the consumers as primitive concepts. The assumptions made by Debreu are implied by the assumptions made in Theorem 3.4.

Theorem 3.4

Let $(X^i,u^i)_{i\in I_M}$ *satisfy the Assumptions A1-A2. Then there exists a set* Ω^1 *open in* Ω *such that* $\Omega \setminus \Omega^1$ *has Lebesgue measure zero and, for every* $\omega \in \Omega^1$, *the number of Walrasian equilibria* (p^*,x^*) *with* $p_N^* = 1$ *of the economy* $\mathcal{E} = ((X^i,u^i,\omega^i)_{i\in I_M})$ *is finite. Moreover, for every* $\overline{\omega} \in \Omega^1$, *there exists a set* O *being open in* Ω *and containing* $\overline{\omega}$, *and there exist continuous functions* $f^k : O \to \mathbb{R}_{++}^N \times X$, $\forall k \in I_{k(\overline{\omega})}$, *for some* $k(\overline{\omega}) \in \mathbb{N}$, *such that, for every*

$\omega \in O$, $f^1(\omega), \dots, f^{k(\overline{\omega})}(\omega)$ *are all the different Walrasian equilibria* $(p(\omega,k), x(\omega,k))_{k\in I_{k(\overline{\omega})}}$ *with* $p(\omega,k)_N = 1$, $\forall k \in I_{k(\overline{\omega})}$, *of the economy* $\mathcal{E} = ((X^i, u^i, \omega^i)_{i\in I_M})$.
See Debreu (1970), Theorem, page 388, and Remark, page 390.

The following extension of Theorem 3.4 is due to Smale (1974).

Theorem 3.5
Let $(X^i)_{i\in I_M}$ *satisfy Assumption A1. Then there exists an open and dense set* $\mathcal{U}^2$ *in* $U \times \Omega$ *such that, for every* $(u,\omega) \in \mathcal{U}^2$, *the number of Walrasian equilibria* (p^*, x^*) *with* $p^*_N = 1$ *of the economy* $\mathcal{E} = ((X^i, u^i, \omega^i)_{i\in I_M})$ *is finite. Moreover, for every* $(\overline{u},\overline{\omega}) \in \mathcal{U}^2$, *there exists a set* $\mathcal{O}$ *being open in* $U \times \Omega$ *and containing* $(\overline{u},\overline{\omega})$, *and there exist continuous functions* $f^k : \mathcal{O} \to \mathbb{R}^N_{++} \times X$, $\forall k \in I_{k(\overline{u},\overline{\omega})}$, *for some* $k(\overline{u},\overline{\omega}) \in \mathbb{N}$, *such that, for every* $(u,\omega) \in \mathcal{O}$, $f^1(u,\omega), \dots, f^{k(u,\omega)}(u,\omega)$ *are all the different Walrasian equilibria* $(p(u,\omega,k), x(u,\omega,k))_{k\in I_{k(\overline{u},\overline{\omega})}}$ *with* $p(u,\omega,k)_N = 1$, $\forall k \in I_{k(\overline{u},\overline{\omega})}$, *of the economy* $\mathcal{E} = ((X^i, u^i, \omega^i)_{i\in I_M})$.
See Smale (1974), Theorem 1, page 3, and Proposition 4, page 7.

The following theorem states that in every Walrasian equilibrium of a typical economy it holds that on every market there is exactly one consumer having the minimal and exactly one consumer having the maximal excess demand in the sense as mentioned previously. A similar result is shown in Laroque (1978) for the uniform rationing system.

Theorem 3.6
Let $(X^i, u^i)_{i\in I_M}, (\tilde{l}, \tilde{L})$ *satisfy the Assumptions A1-A2 and A4. Then there exists a subset* Ω^2 *of* Ω *such that* $\Omega \setminus \Omega^2$ *has Lebesgue measure zero and, for every* $\omega \in \Omega^2$, *for every Walrasian equilibrium* (p^*, x^*) *with* $p^*_N = 1$ *of the economy* $\mathcal{E} = ((X^i, u^i, \omega^i)_{i\in I_M})$, *for every* $j \in I_{N-1}$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$.
Proof
If $M = 1$, then the proof is trivial, so assume $M \geq 2$ for the remainder of the proof. It is easily seen that Assumption A4 guarantees that, for every $\omega \in \Omega$, for every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, \preceq^i, \omega^i)_{i\in I_M})$, $\#\underline{I}_j(x^*) \geq 1$, $\forall j \in I_{N-1}$, and $\#\overline{I}_j(x^*) \geq 1$, $\forall j \in I_{N-1}$. Moreover, it follows easily that every Walrasian equilibrium price system is strictly positive. Let some $j' \in I_{N-1}$, $\overline{\omega} \in \Omega$, and $(p', \overline{x}) \in \mathbb{R}^N_{++} \times X$ with $p'_N = 1$ be given. If $(p', \overline{x})$ is a Walrasian equilibrium of the economy $\mathcal{E} = ((X^i, u^i, \overline{\omega}^i)_{i\in I_M})$ and $\#\underline{I}_{j'}(\overline{x}) > 1$, then there exists $i^1, i^2 \in I_M$ with $i^1 \neq i^2$, $\overline{\lambda}^i \in \mathbb{R}$, $\forall i \in I_M$, and $\overline{q}_{j'} \in [0,1]$ such that

$$\begin{aligned}
\partial_{x^i} u^i(\overline{x}^i)^\top - \overline{\lambda}^i(p'_1, \dots, p'_{N-1}, 1)^\top &= 0^N, \ \forall i \in I_M, & (1)\\
(p'_1, \dots, p'_{N-1}, 1)\overline{x}^i - (p'_1, \dots, p'_{N-1}, 1)\overline{\omega}^i &= 0, \quad \forall i \in I_M, & (2)\\
\textstyle\sum_{i\in I_M} \overline{x}^i_j - \sum_{i\in I_M} \overline{\omega}^i_j &= 0, \quad \forall j \in I_{N-1}, & (3)\\
\overline{x}^i_{j'} - \overline{\omega}^i_{j'} - \tilde{l}^i_{j'}(\overline{q}) &= 0, \quad \forall i \in \{i^1, i^2\}, & (4)
\end{aligned}$$

with $\overline{q}_1, \ldots, \overline{q}_{j'-1}, \overline{q}_{j'+1}, \ldots, \overline{q}_N$ arbitrarily given elements of $[0,1]$. Notice that in (3) the condition that on the market of the numeraire commodity the total excess demand is equal to zero is not specified. This condition is implied by the equations in (2) and (3).
Since the function $\tilde{l}$ is assumed to be continuously differentiable and $\partial_{q_{j'}}\tilde{l}_{j'}(\overline{q}) \neq 0^M$, $\forall \overline{q} \in Q^N$, there exists a continuously differentiable extension of $\tilde{l}$, also denoted by $\tilde{l}$, satisfying $\partial_{q_{j'}}\tilde{l}_{j'}(\overline{q}) \neq 0^M$ when $\overline{q}_{j'} \in (-\varepsilon, 1+\varepsilon)$ and $\overline{q}_j \in [0,1]$, $\forall j \in I_N \setminus \{j'\}$. The function

$$\psi : X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times \Omega \times (-\varepsilon, 1+\varepsilon) \to \mathbb{R}^{MN+M+N+1}$$

is defined such that, for every $(x, \lambda, (p_1, \ldots, p_{N-1})^\top, \omega, q_{j'}) \in X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times \Omega \times (-\varepsilon, 1+\varepsilon)$, $\psi(x, \lambda, (p_1, \ldots, p_{N-1})^\top, \omega, q_{j'})$ is given by the left-hand side of (1)-(4). For every $\omega \in \Omega$, the function

$$\psi^\omega : X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times (-\varepsilon, 1+\varepsilon) \to \mathbb{R}^{MN+M+N+1}$$

is defined by associating with every $(x, \lambda, (p_1, \ldots, p_{N-1})^\top, q_{j'}) \in X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times (-\varepsilon, 1+\varepsilon)$ the element $\psi^\omega(x, \lambda, (p_1, \ldots, p_{N-1})^\top, q_{j'}) = \psi(x, \lambda, (p_1, \ldots, p_{N-1})^\top, \omega, q_{j'})$.
Let $\overline{\xi} = (\overline{x}, \overline{\lambda}, (p'_1, \ldots, p'_{N-1})^\top, \overline{\omega}, \overline{q}_{j'}) \in X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times \Omega \times (-\varepsilon, 1+\varepsilon)$ be such that $\psi(\overline{\xi}) = 0^{MN+M+N+1}$ The matrix of partial derivatives of ψ evaluated at $\overline{\xi}$ is denoted by $\overline{M}$ and given in Table 3.1. Again, $\overline{q}_1, \ldots, \overline{q}_{j'-1}, \overline{q}_{j'+1}, \ldots, \overline{q}_N$ are arbitrarily chosen elements of $[0,1]$. It will be shown that the matrix $\overline{M}$ has rank $MN+M+N+1$. Let $y \in \mathbb{R}^{MN+M+N+1}$ be such that $y^\top \overline{M} = 0^{2MN+M+N^\top}$. Then, $y^\top \partial_{\omega^i_N}\psi(\overline{\xi}) = 0$, $\forall i \in I_M$, implies

$$y_{MN+i} = 0, \ \forall i \in I_M. \tag{5}$$

Moreover, (5) and $y^\top \partial_{\omega^1_j}\psi(\overline{\xi}) = 0$, $\forall j \in I_{N-1} \setminus \{j'\}$, implies

$$y_{MN+M+j} = 0, \ \forall j \in I_{N-1} \setminus \{j'\}. \tag{6}$$

From (5) and (6) it follows that

$$\begin{aligned}
y^\top \partial_{\omega^{i^1}_{j'}}\psi(\overline{\xi}) &= -y_{MN+M+j'} - y_{MN+M+N} = 0, & (7)\\
y^\top \partial_{\omega^{i^2}_{j'}}\psi(\overline{\xi}) &= -y_{MN+M+j'} - y_{MN+M+N+1} = 0, & (8)\\
y^\top \partial_{\omega^{i}_{j'}}\psi(\overline{\xi}) &= -y_{MN+M+j'} = 0, \ \forall i \in I_M \setminus \{i^1, i^2\}, & (9)\\
y^\top \partial_{q_{j'}}\psi(\overline{\xi}) &= -\partial_{q_{j'}}\tilde{l}^{i^1}_{j'}(\overline{q}) y_{MN+M+N} - \partial_{q_{j'}}\tilde{l}^{i^2}_{j'}(\overline{q}) y_{MN+M+N+1} = 0. & (10)
\end{aligned}$$

If $M \geq 3$, then (9) implies $y_{MN+M+j'} = 0$, so then it follows from (7) and (8) that $y_{MN+M+N} = y_{MN+M+N+1} = 0$. If $M = 2$, then (7) and (8) implies $y_{MN+M+N} = y_{MN+M+N+1} = -y_{MN+M+j'}$. Then it follows from (10) that

$$(\partial_{q_{j'}}\tilde{l}^{i^1}_{j'}(\overline{q}) + \partial_{q_{j'}}\tilde{l}^{i^2}_{j'}(\overline{q})) y_{MN+M+j'} = 0,$$

	MN	M	$N-1$	MN	1
MN	$\begin{matrix} \partial^2_{x^1x^1}u^1(\overline{x}^1) & & 0 \\ & \ddots & \\ 0 & & \partial^2_{x^Mx^M}u^M(\overline{x}^M) \end{matrix}$	$\begin{matrix} \begin{matrix} -p'_1 \\ \vdots \\ -p'_{N-1} \\ -1 \end{matrix} & & 0 \\ & \ddots & \\ 0 & & \begin{matrix} -p'_1 \\ \vdots \\ -p'_{N-1} \\ -1 \end{matrix} \end{matrix}$	$\begin{matrix} -\overline{\lambda}^1 I^{N-1} \\ 0^{N-1\top} \\ \vdots \\ -\overline{\lambda}^M I^{N-1} \\ 0^{N-1\top} \end{matrix}$	$0^{MN\times MN}$	0^{MN}
M	$\begin{matrix} (p'_j)_{j\in I_{N-1}}\ 1 & & 0 \\ & \ddots & \\ 0 & & (p'_j)_{j\in I_{N-1}}\ 1 \end{matrix}$	$0^{M\times M}$	$\begin{matrix} (\overline{x}^1_j-\overline{\omega}^1_j)_{j\in I_{N-1}} \\ \vdots \\ (\overline{x}^M_j-\overline{\omega}^M_j)_{j\in I_{N-1}} \end{matrix}$	$\begin{matrix} (-p'_j)_{j\in I_{N-1}}\ -1 & & 0 \\ & \ddots & \\ 0 & & (-p'_j)_{j\in I_{N-1}}\ -1 \end{matrix}$	0^{M}
$N-1$	$I^{N-1}\ 0^{N-1}\ \dots\ I^{N-1}\ 0^{N-1}$	$0^{(N-1)\times M}$	$0^{(N-1)\times(N-1)}$	$-I^{N-1}\ 0^{N-1}\ \dots\ -I^{N-1}\ 0^{N-1}$	0^{N-1}
1	$e^{MN}((i^1-1)N+j')^\top$	$0^{M\top}$	$0^{N-1\top}$	$-e^{MN}((i^1-1)N+j')^\top$	$-\partial_{q_{j'}}\widetilde{l}^{i^1}_{j'}(\overline{q})$
1	$e^{MN}((i^2-1)N+j')^\top$	$0^{M\top}$	$0^{N-1\top}$	$-e^{MN}((i^2-1)N+j')^\top$	$-\partial_{q_{j'}}\widetilde{l}^{i^2}_{j'}(\overline{q})$

Table 3.1. The matrix $\overline{M}$.

so $y_{MN+M+j'}=0$ since $\partial_{q_{j'}}\widetilde{l}_{j'}(\overline{q})\neq(0,0)$ by Assumption A4. Therefore,

$$y_{MN+M+j'}=0,\ y_{MN+M+N}=0,\ \text{and}\ y_{MN+M+N+1}=0. \tag{11}$$

Using the non-zero Gaussian curvature of $\preceq^i$, $\forall i\in I_M$, it follows that

$$\det\left(\begin{bmatrix} \partial^2_{x^ix^i}u^i(\overline{x}^i) & \partial_{x^i}u^i(\overline{x}^i)^\top \\ \partial_{x^i}u^i(\overline{x}^i) & 0 \end{bmatrix}\right)\neq 0,\ \forall i\in I_M.$$

So the first N rows of this matrix are independent. Since $\partial_{x^i}u^i(\overline{x}^i)=\overline{\lambda}^i(p'_1,\dots,p'_{N-1},1)$, $\forall i\in I_M$, it follows that the N rows of the matrix

$$\left[\ \partial^2_{x^ix^i}u^i(\overline{x}^i)\quad(-p'_1,\dots,-p'_{N-1},-1)^\top\ \right]$$

are independent. Since, by (5), (6), and (11),

$$y^\top\partial_{x^i}\psi(\overline{\xi})=\sum_{j\in I_N}\partial_{x^i}\psi_{(i-1)N+j}(\overline{\xi})y_{(i-1)N+j}=0^{N^\top},\ \forall i\in I_M,$$

it follows that, for every $i \in I_M$,

$$y_{(i-1)N+j} = 0,\ \forall j \in I_N.$$

Therefore, $y = 0^{MN+M+N+1}$, so $\overline{M}$ has rank $MN+M+N+1$, and ψ intersects $\{0^{MN+M+N+1}\}$ transversally, $\psi \pitchfork \{0^{MN+M+N+1}\}$. Let the set $\overline{\Omega}_{j'}$ be defined by

$$\overline{\Omega}_{j'} = \{\omega \in \Omega \mid \psi^{\omega} \pitchfork \{0^{MN+M+N+1}\}\}.$$

Since $X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times \Omega \times (-\varepsilon, 1+\varepsilon)$ is an $(2MN+M+N)$-dimensional C^∞ manifold, $\mathbb{R}^{MN+M+N+1}$ is an $(MN+M+N+1)$-dimensional C^∞ manifold, $0^{MN+M+N+1}$ is a 0-dimensional C^∞ manifold, and $\psi \in C^1(X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times \Omega \times (-\varepsilon, 1+\varepsilon), \mathbb{R}^{MN+M+N+1})$, it follows from the transversality theorem, see for instance Theorem I.2.2 of Mas-Colell (1985), that the set $\Omega \setminus \overline{\Omega}_{j'}$ has Lebesgue measure zero. For every $\omega \in \overline{\Omega}_{j'}$, ψ^{ω} is a function from an $(MN+M+N)$-dimensional C^∞ manifold into an $(MN+M+N+1)$-dimensional C^∞ manifold, $\{0^{MN+M+N+1}\}$ is a 0-dimensional C^∞ manifold, $\psi^{\omega} \in C^1(X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times (-\varepsilon, 1+\varepsilon), \mathbb{R}^{MN+M+N+1})$, and $\psi^{\omega} \pitchfork \{0^{MN+M+N+1}\}$, so ${\psi^{\omega}}^{-1}(\{0^{MN+N+M+1}\}) = \emptyset$. Hence, the set of initial endowments ω of Ω such that there exists a Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ and $\#\underline{I}_{j'}(x^*) > 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ has Lebesgue measure zero. Similarly, it can be shown the set of initial endowments ω of Ω such that there exists a Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ and $\#\overline{I}_{j'}(x^*) > 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ has Lebesgue measure zero. Since a finite union of sets with Lebesgue measure zero has Lebesgue measure zero, it follows that, for every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$, for every $j \in I_N$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$, except for a set of initial endowments ω of Ω having Lebesgue measure zero. Q.E.D.

The results of Theorem 3.3, Theorem 3.5, and Theorem 3.6 yield the following theorem.

Theorem 3.7
Let $(X^i)_{i \in I_M}, (\tilde{l}, \tilde{L})$ satisfy the Assumptions A1 and A4. Then there exists an open and dense set $\mathcal{U}^3$ in $U \times \Omega$ such that, for every $(u, \omega) \in \mathcal{U}^3$, every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ is locally unique and regular, while, for every $j \in I_N$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$. Moreover, for every $(\overline{u}, \overline{\omega}) \in \mathcal{U}^3$, there exists a set $\mathcal{O}$ being open in $U \times \Omega$ and containing $(\overline{u}, \overline{\omega})$, and there exist continuous functions $f^k : \mathcal{O} \to \mathbb{R}^N_{++} \times X$, $\forall k \in I_{k(\overline{u},\overline{\omega})}$, for some $k(\overline{u}, \overline{\omega}) \in \mathbf{N}$, such that, for every $(u, \omega) \in \mathcal{O}$, $f^1(u, \omega), \ldots, f^{k(\overline{u},\overline{\omega})}(u, \omega)$ are all the different Walrasian equilibria $(p(u, \omega, k), x(u, \omega, k))_{k \in I_{k(\overline{u},\overline{\omega})}}$ with $p(u, \omega, k)_N = 1$, $\forall k \in I_{k(\overline{u},\overline{\omega})}$, of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$.

Proof

Let $\mathcal{U}^1$ and $\mathcal{U}^2$ denote the sets given in Theorem 3.3 and Theorem 3.5, respectively. Clearly, the set $\mathcal{U}^1 \cap \mathcal{U}^2$ is open and dense in $U \times \Omega$ being an intersection of two open and dense sets. Let $\mathcal{U}^3$ be the set of utility functions and initial endowments (u, ω) of $\mathcal{U}^1 \cap \mathcal{U}^2$ such that, for every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$, for every $j \in I_{N-1}$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$. It will be shown that $\mathcal{U}^3$ is open and dense in $U \times \Omega$. Notice that this is trivial if $M = 1$, so assume $M \geq 2$ for the remainder of the proof.

For every $u \in U$, let Ω_u denote the set of initial endowments such that, for every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$, for every $j \in I_{N-1}$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$. Obviously, by Theorem 3.6, the set $\{(u, \omega) \in U \times \Omega \mid \omega \in \Omega_u\}$ is dense in $U \times \Omega$. Now $\mathcal{U}^1 \cap \mathcal{U}^2 \cap \{(u, \omega) \in U \times \Omega \mid \omega \in \Omega_u\}$ is dense in $U \times \Omega$ as an intersection of an open and dense sets in $U \times \Omega$ and a dense set in $U \times \Omega$. Since $\mathcal{U}^1 \cap \mathcal{U}^2 \cap \{(u, \omega) \in U \times \Omega \mid \omega \in \Omega_u\} = \mathcal{U}^3$, it follows that $\mathcal{U}^3$ is dense in $U \times \Omega$.

Let some $(\overline{u}, \overline{\omega}) \in \mathcal{U}^3$ be given and let $f^k : \mathcal{O} \to \mathbb{R}^N_{++} \times X$, $\forall k \in I_{k(\overline{u}, \overline{\omega})}$, be the continuous functions given in Theorem 3.5. For every $j \in I_{N-1}$, for every $k \in I_{k(\overline{u}, \overline{\omega})}$, denote the elements $\underline{q}(x(\overline{u}, \overline{\omega}, k))$ and $\overline{q}(x(\overline{u}, \overline{\omega}, k))$, and the sets $\underline{I}_j(x(\overline{u}, \overline{\omega}, k))$ and $\overline{I}_j(x(\overline{u}, \overline{\omega}, k))$, corresponding to the Walrasian equilibrium $f^k(\overline{u}, \overline{\omega}) = (p(\overline{u}, \overline{\omega}, k), x(\overline{u}, \overline{\omega}, k))$, by $\underline{q}^k$, $\overline{q}^k$, $\underline{I}^k_j$, and $\overline{I}^k_j$, respectively. Let the function $f^0 : \mathcal{O} \to \Omega$ be defined by $f^0(u, \omega) = \omega$, $\forall (u, \omega) \in \mathcal{O}$. Clearly, f^0 is continuous. For every $k \in I_{k(\overline{u}, \overline{\omega})}$, for every $j \in I_{N-1}$, let the function $g^k_j : \mathbb{R}^N_{++} \times X \times \Omega \to \mathbb{R}$ be defined by associating with every $(p, x, \omega) \in \mathbb{R}^N_{++} \times X \times \Omega$ the element

$$g^k_j(p, x, \omega) = \min\left(\left\{x^i_j - \omega^i_j - \widehat{l}^i_j(\underline{q}^k) \middle| \, i \in I_M \setminus \underline{I}^k_j\right\} \cup \left\{\widehat{L}^i_j(\overline{q}^k) - x^i_j + \omega^i_j \middle| \, i \in I_M \setminus \overline{I}^k_j\right\}\right).$$

Clearly, g^k_j is continuous. Let the function $h : \mathcal{O} \to \mathbb{R}$ be defined by

$$h(u, \omega) = \min\left(\left\{g^k_j(f^k(u, \omega), f^0(u, \omega)) \middle| \, j \in I_{N-1}, \; k \in I_{k(\overline{u}, \overline{\omega})}\right\}\right), \; \forall (u, \omega) \in \mathcal{O}.$$

Clearly, h is continuous. Notice that $h(\overline{u}, \overline{\omega}) > 0$. Since $(\widetilde{l}, \widetilde{L})$ is monotonic and continuous, there exists $\varepsilon \in \mathbb{R}_{++}$ such that, for every $(u, \omega) \in \mathcal{O}$ with $|h(\overline{u}, \overline{\omega}) - h(u, \omega)| < \varepsilon$, for every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$, for every $j \in I_{N-1}$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$. The set $h^{-1}((h(\overline{u}, \overline{\omega}) - \varepsilon, h(\overline{u}, \overline{\omega}) + \varepsilon))$ is open in $\mathcal{O}$ by the continuity of h, hence also open in $U \times \Omega$, and contains $(\overline{u}, \overline{\omega})$. Therefore, $\mathcal{U}^3$ is open in $U \times \Omega$. Q.E.D.

For the remainder of this section, let $(X^i, u^i, \omega^i)_{i \in I_M}$, $(\widetilde{l}, \widetilde{L})$, $((\pi^{ik})_{i \in I_M})_{k \in I_2}$ be given such that the Assumptions A1-A4 and A6 are satisfied and (u, ω) is an element of the set $\mathcal{U}^3$ given in Theorem 3.7.

Let a Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ be given. From Theorem 3.7 it follows that (p^*, x^*) is a locally unique and regular Walrasian equilibrium of the economy $\mathcal{E}$, while, for every $j \in I_{N-1}$, $\underline{I}_j(x^*) = \overline{I}_j(x^*) = 1$. From the remark below Theorem 3.2 it follows that, for every price system $p \in \overline{P}$ being close to p^*, there exists a uniquely determined Drèze equilibrium of the economy $\widetilde{\mathcal{E}}(p,p)$ being close to (p^*, x^*). This suggests a very appealing alternative model for the competition of votes between political candidates. Consider the Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E}$ as the status quo of the economy. The political candidates can either choose a direction of change with respect to the status quo, or choose to stay at the status quo. If a political candidate chooses a specific direction of change, then it is assumed that the uniquely determined Drèze equilibrium specified in Theorem 3.2 being close to the Walrasian equilibrium results. So, political candidates are no longer assumed to choose a specific Drèze equilibrium, but are restricted to choose between local options. Moreover, from Theorem 3.2 it follows that the change in utility of the consumers is uniquely determined if a political candidate chooses a specific direction of change. It is again assumed that political candidates maximize their expected plurality in the elections, or, more precisely, political candidates choose a direction of change affecting their marginal expected plurality optimally. Notice that this way of modelling the political system captures some interesting real world phenomena. First of all, the cost of acquiring information for political candidates is usually very high for actions far removed from the status quo. Moreover, institutional reasons and commitments made in the past often require political candidates to stay near the status quo. Finally, political candidates need only bother about the price regulations to implement, they no longer need to specify the resulting Drèze equilibrium if there is more than one.

Let a Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E}$ be the status quo. The set of admissible actions $\widetilde{\mathcal{A}}^k$ of a political political candidate $k \in I_2$ is given by a subset of the set $\widetilde{\mathcal{A}}$, defined by

$$\widetilde{\mathcal{A}} = \left\{ \tilde{p} \in \mathbb{R}^{N-1} \middle| \, \|\tilde{p}\|_2 = 1 \right\} \cup \left\{ 0^{N-1} \right\}.$$

The action $\tilde{p}^k \in \widetilde{\mathcal{A}}^k$ of a political candidate $k \in I_2$ corresponds to a change of $(p^*_1, \ldots, p^*_{N-1})^\top$ in the direction $\tilde{p}^k$. If $\tilde{p}^k = 0^{N-1}$, then political candidate $k \in I_2$ chooses to stay at the status quo (p^*, x^*). For the main results, the following very weak assumption is made.

A8. For every political candidate $k \in I_2$, the set of admissible actions $\widetilde{\mathcal{A}}^k$ is non-empty, closed, and $\widetilde{\mathcal{A}}^k \subset \widetilde{\mathcal{A}}$.

The pay-off function $\widetilde{K}^1 : \widetilde{\mathcal{A}}^1 \times \widetilde{\mathcal{A}}^2 \to \mathbb{R}$ of political candidate 1 is defined by associating with every $(\tilde{p}^1, \tilde{p}^2) \in \widetilde{\mathcal{A}}^1 \times \widetilde{\mathcal{A}}^2$ the element

$$\begin{aligned}
&\widetilde{K}^1(\tilde{p}^1, \tilde{p}^2) \\
&= \sum_{i \in I_M} \partial_{v^1} \pi^{i1} \left(\widehat{v}^i(p^*_1, \ldots, p^*_{N-1}), \widehat{v}^i(p^*_1, \ldots, p^*_{N-1}) \right) \partial_{(p_1, \ldots, p_{N-1})^\top} \widehat{v}^i(p^*_1, \ldots, p^*_{N-1}) \tilde{p}^1
\end{aligned}$$

$$
\begin{aligned}
&- \sum_{i\in I_M} \partial_{v^1}\pi^{i2}\left(\widehat{v}^i(p_1^*,\dots,p_{N-1}^*),\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\right)\partial_{(p_1,\dots,p_{N-1})^\top}\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\widetilde{p}^1 \\
&+ \sum_{i\in I_M} \partial_{v^2}\pi^{i1}\left(\widehat{v}^i(p_1^*,\dots,p_{N-1}^*),\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\right)\partial_{(p_1,\dots,p_{N-1})^\top}\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\widetilde{p}^2 \\
&- \sum_{i\in I_M} \partial_{v^2}\pi^{i2}\left(\widehat{v}^i(p_1^*,\dots,p_{N-1}^*),\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\right)\partial_{(p_1,\dots,p_{N-1})^\top}\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\widetilde{p}^2.
\end{aligned}
$$

The pay-off function $\widetilde{K}^2 : \widetilde{\mathcal{A}}^1 \times \widetilde{\mathcal{A}}^2 \to \mathbb{R}$ of political candidate 2 is easily seen to be given by $\widetilde{K}^2 = -\widetilde{K}^1$.

Definition 3.8 (Directional Political Economic Equilibrium)
A directional political economic equilibrium of the political economic system $\overline{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i\in I_M}, (\widetilde{l}, \widetilde{L}), (\widetilde{\mathcal{A}}^k, (\pi^{ik})_{i\in I_M})_{k\in I_2})$ *with status quo the locally unique, regular Walrasian equilibrium* (p^*, x^*) *with* $p_N^* = 1$ *and* $\underline{l}_j(x^*) = \overline{l}_j(x^*) = 1$, $\forall j \in I_{N-1}$, *of the economy* $\mathcal{E} = (X^i, u^i, \omega^i)_{i\in I_M}$), *is a Nash equilibrium of the mixed extension of the game* $\widetilde{\mathcal{G}} = (\widetilde{\mathcal{A}}^1, \widetilde{\mathcal{A}}^2, \widetilde{K}^1, \widetilde{K}^2)$.

Let a Walrasian equilibrium (p^*, x^*) with $p_N^* = 1$ of the economy $\mathcal{E}$ be the status quo. For every $k \in I_2$, let $k' \in I_2$ be such that $k' \neq k$, and define the function $\widetilde{K}^{k+} : \widetilde{\mathcal{A}}^k \to \mathbb{R}$ by associating with every $\widetilde{p}^k \in \widetilde{\mathcal{A}}^k$

$$
\begin{aligned}
\widetilde{K}^{k+}(\widetilde{p}^k) = &\sum_{i\in I_M} \partial_{v^k}\pi^{ik}\left(\widehat{v}^i(p_1^*,\dots,p_{N-1}^*),\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\right)\partial_{(p_1,\dots,p_{N-1})^\top}\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\widetilde{p}^k \\
&-\sum_{i\in I_M} \partial_{v^k}\pi^{ik'}\left(\widehat{v}^i(p_1^*,\dots,p_{N-1}^*),\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\right)\partial_{(p_1,\dots,p_{N-1})^\top}\widehat{v}^i(p_1^*,\dots,p_{N-1}^*)\widetilde{p}^k.
\end{aligned}
$$

Now the following theorem can be shown.

Theorem 3.9
Let the political economic system $\overline{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i\in I_M}, (\widetilde{l}, \widetilde{L}), (\widetilde{\mathcal{A}}^k, (\pi^{ik})_{i\in I_M})_{k\in I_2})$ *with status quo* (p^*, x^*) *satisfy the Assumptions A1-A4, A6, and A8, where* (u, ω) *is an element of the set* $\mathcal{U}^3$ *given in Theorem 3.7 and* (p^*, x^*) *with* $p_N^* = 1$ *is a Walrasian equilibrium of the economy* $\mathcal{E} = ((X^i, u^i, \omega^i)_{i\in I_M})$. *Then there exists a directional political economic equilibrium in pure strategies of the political economic system* $\overline{\mathcal{E}}$ *with status quo* (p^*, x^*). *Moreover,* $(\widetilde{p}^{*1}, \widetilde{p}^{*2}) \in \widetilde{\mathcal{A}}^1 \times \widetilde{\mathcal{A}}^2$ *is a directional political economic equilibrium in pure strategies of the political economic system* $\overline{\mathcal{E}}$ *with status quo* (p^*, x^*) *if and only if*

$$
\begin{aligned}
\widetilde{K}^{1+}(\widetilde{p}^{*1}) &= \max\left(\left\{\widetilde{K}^{1+}(\widetilde{p}^1)\,\middle|\,\widetilde{p}^1 \in \widetilde{\mathcal{A}}^1\right\}\right), \\
\widetilde{K}^{2+}(\widetilde{p}^{*2}) &= \max\left(\left\{\widetilde{K}^{2+}(\widetilde{p}^2)\,\middle|\,\widetilde{p}^2 \in \widetilde{\mathcal{A}}^2\right\}\right).
\end{aligned}
$$

Proof
For every $k \in I_2$, the set $\widetilde{\mathcal{A}}^k$ is compact and the function $\widetilde{K}^{k+}$ is continuous, so there exists $\widetilde{p}^{*k} \in \widetilde{\mathcal{A}}^k$ such that

$$
\widetilde{K}^{k+}(\widetilde{p}^{*k}) = \max\left(\left\{\widetilde{K}^{k+}(\widetilde{p}^k)\,\middle|\,\widetilde{p}^k \in \widetilde{\mathcal{A}}^k\right\}\right).
$$

Clearly,

$$\begin{aligned}
\widetilde{K}^1(\tilde{p}^{*1},\tilde{p}^{*2}) &= \widetilde{K}^{1+}(\tilde{p}^{*1}) - \widetilde{K}^{2+}(\tilde{p}^{*2}) \geq \widetilde{K}^{1+}(\tilde{p}^{1}) - \widetilde{K}^{2+}(\tilde{p}^{*2}) \\
&= \widetilde{K}^1(\tilde{p}^{1},\tilde{p}^{*2}),\ \forall \tilde{p}^1 \in \tilde{\mathcal{A}}^1, \\
\widetilde{K}^2(\tilde{p}^{*1},\tilde{p}^{*2}) &= \widetilde{K}^{2+}(\tilde{p}^{*2}) - \widetilde{K}^{1+}(\tilde{p}^{*1}) \geq \widetilde{K}^{2+}(\tilde{p}^{2}) - \widetilde{K}^{1+}(\tilde{p}^{*1}) \\
&= \widetilde{K}^2(\tilde{p}^{*1},\tilde{p}^{2}),\ \forall \tilde{p}^2 \in \tilde{\mathcal{A}}^2.
\end{aligned}$$

So, $(\tilde{p}^{*1},\tilde{p}^{*2})$ is a directional political economic equilibrium in pure strategies of the political economic system $\overline{\mathcal{E}}$ with status quo (p^*,x^*).
Let $(\tilde{p}^{*1},\tilde{p}^{*2}) \in \tilde{\mathcal{A}}^1 \times \tilde{\mathcal{A}}^2$ be any political economic equilibrium in pure strategies of the economy $\overline{\mathcal{E}}$ with status quo (p^*,x^*). Then,

$$\begin{aligned}
\widetilde{K}^1(\tilde{p}^{*1},\tilde{p}^{*2}) &= \widetilde{K}^{1+}(\tilde{p}^{*1}) - \widetilde{K}^{2+}(\tilde{p}^{*2}) \geq \widetilde{K}^1(\tilde{p}^{1},\tilde{p}^{*2}) \\
&= \widetilde{K}^{1+}(\tilde{p}^{1}) - \widetilde{K}^{2+}(\tilde{p}^{*2}),\ \forall \tilde{p}^1 \in \tilde{\mathcal{A}}^1, \\
\widetilde{K}^2(\tilde{p}^{*1},\tilde{p}^{*2}) &= \widetilde{K}^{2+}(\tilde{p}^{*2}) - \widetilde{K}^{1+}(\tilde{p}^{*1}) \geq \widetilde{K}^2(\tilde{p}^{*1},\tilde{p}^{2}) \\
&= \widetilde{K}^{2+}(\tilde{p}^{2}) - \widetilde{K}^{1+}(\tilde{p}^{*1}),\ \forall \tilde{p}^2 \in \tilde{\mathcal{A}}^2.
\end{aligned}$$

So, $\widetilde{K}^{1+}(\tilde{p}^{*1}) = \max(\{\widetilde{K}^{1+}(\tilde{p}^1) \mid \tilde{p}^1 \in \tilde{\mathcal{A}}^1\})$ and $\widetilde{K}^{2+}(\tilde{p}^{*2}) = \max(\{\widetilde{K}^{2+}(\tilde{p}^2) \mid \tilde{p}^2 \in \tilde{\mathcal{A}}^2\})$.
Q.E.D.

4 Generically Chosen Price Regulations

The existence of a political economic equilibrium of the political economic system $\hat{\mathcal{E}}$ has been shown in Herings (1994). In the example of Herings (1994) price regulations incompatible with the Walrasian equilibrium price system were chosen by the political candidates in the political economic equilibrium. The existence of a directional political economic equilibrium of the political economic system $\overline{\mathcal{E}}$ with status quo a Walrasian equilibrium has been shown in Section 3. In the example of Section 5 the political candidates will choose directions of movement away from the Walrasian equilibrium price system in the directional political economic equilibrium. Nevertheless, it is not clear whether this is the typical case. To answer this question, the following assumption with respect to the model of Section 3 will be made in the main results of this section.

A9. For every political candidate $k \in I_2$, $\tilde{\mathcal{A}}^k = \tilde{\mathcal{A}}$.

Making Assumption A9 and using Theorem 3.9 it is possible to show the even stronger result of Theorem 4.1, where the directional political economic equilibria in pure strategies of $\overline{\mathcal{E}}$ with status quo (p^*,x^*) are characterized.

Let $(X^i,u^i,\omega^i)_{i\in I_M},(\bar{l},\tilde{L}),((\pi^{ik})_{i\in I_M})_{k\in I_2}$ be given such that the Assumptions A1-A4 and A6 are satisfied and (u,ω) is an element of the set $\mathcal{U}^3$ given in Theorem 3.7. Moreover, let a Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ be given. For every $k \in I_2$, let $k' \in I_2$ be such that $k' \neq k$, and define the vector $\overline{K}^k \in \mathbb{R}^{N-1}$ by

$$\begin{aligned} \overline{K}^k \; = \; & \Bigg(\sum_{i \in I_M} \partial_{v^k} \pi^{ik} \left(\widehat{v}^i(p^*_1, \ldots, p^*_{N-1}), \widehat{v}^i(p^*_1, \ldots, p^*_{N-1})\right) \partial_{(p_1, \ldots, p_{N-1})^\top} \widehat{v}^i(p^*_1, \ldots, p^*_{N-1}) \\ & - \sum_{i \in I_M} \partial_{v^k} \pi^{ik'} \left(\widehat{v}^i(p^*_1, \ldots, p^*_{N-1}), \widehat{v}^i(p^*_1, \ldots, p^*_{N-1})\right) \partial_{(p_1, \ldots, p_{N-1})^\top} \widehat{v}^i(p^*_1, \ldots, p^*_{N-1}) \Bigg)^\top . \end{aligned}$$

Theorem 4.1

Let the political economic system $\overline{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_M}, (\widetilde{l}, \widetilde{L}), (\widetilde{\mathcal{A}}^k, (\pi^{ik})_{i \in I_M})_{k \in I_2})$ with status quo (p^*, x^*) satisfy the Assumptions A1-A4, A6, and A9, where (u, ω) is an element of the set $\mathcal{U}^3$ given in Theorem 3.7 and (p^*, x^*) with $p^*_N = 1$ is a Walrasian equilibrium of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$. Let $(\widetilde{p}^{*1}, \widetilde{p}^{*2})$ be a directional political economic equilibrium of the political economic system $\overline{\mathcal{E}}$ with status quo (p^*, x^*). For every $k \in I_2$, either $\overline{K}^k = 0^{N-1}$, or $\overline{K}^k \neq 0^{N-1}$ and $\widetilde{p}^{*k}_{\cdot} = \overline{K}^k / \|\overline{K}^k\|_2$.

Proof

From Theorem 3.9 it follows that

$$\begin{aligned} \widetilde{K}^{1+}(\widetilde{p}^{*1}) &= \max\left(\left\{\widetilde{K}^{1+}(\widetilde{p}^1) \,\middle|\, \widetilde{p}^1 \in \widetilde{\mathcal{A}}^1\right\}\right), \\ \widetilde{K}^{2+}(\widetilde{p}^{*2}) &= \max\left(\left\{\widetilde{K}^{2+}(\widetilde{p}^2) \,\middle|\, \widetilde{p}^2 \in \widetilde{\mathcal{A}}^2\right\}\right). \end{aligned}$$

For every $k \in I_2$, since $\widetilde{K}^{k+}(\widetilde{p}^k) = \overline{K}^k \cdot \widetilde{p}^k$, $\forall \widetilde{p}^k \in \widetilde{\mathcal{A}}^k$, and since $\widetilde{\mathcal{A}}^k$ is the union of the unit sphere $\widetilde{B}^{N-2}(0^{N-1}, 1)$ and $\{0^{N-1}\}$, the theorem follows immediately. Q.E.D.

If in Theorem 4.1 it holds that $\overline{K}^k = 0^{N-1}$ for a political candidate $k \in I_2$, then, given the action chosen by his opponent, every admissible action yields political candidate k the same pay-off. If both $\overline{K}^1 \neq 0^{N-1}$ and $\overline{K}^2 \neq 0^{N-1}$, then the directional political economic equilibrium is unique.

It could be the case that in a typical political economic equilibrium both political candidates propose a price regulation corresponding to the same Walrasian equilibrium or that in a typical directional political economic equilibrium both political candidates choose to stay at the Walrasian equilibrium, being the status quo. It is clear that it is always possible to construct examples where this happens. Moreover, since it is well-known that Drèze equilibria not corresponding to a Walrasian equilibrium are usually not Pareto efficient, it would not be completely surprising if this would turn out to be the case generically, or at least for a non-degenerate class of economies. Nevertheless, in this section it is shown that the situation in the example of Herings (1994) and in the example to be presented in Section 5 are typical cases. Moreover, it is shown that if the opponent of a political candidate chooses an action corresponding to a Walrasian equilibrium, then the generic case is that the action of the political candidate leading to the same Walrasian equilibrium can be improved upon by playing an action not corresponding to a Walrasian equilibrium. Similarly, it is shown that, generically, political candidates propose directions of movement away from the status quo.

Consider the political economic system $\widehat{\mathcal{E}}$. Suppose political candidate 2 chooses an action $(p^*, p^*, q^*) \in \mathcal{A}^2$ corresponding to a Walrasian equilibrium of the economy $\mathcal{E}$. Then it will be shown that only in degenerate cases the best response of political candidate 1 is also to choose the action (p^*, p^*, q^*). More precisely, it will be shown that, generically, for every open set O in $\mathbb{R}^{N-1}_{++}$ containing $(p_1^*, \ldots, p_{N-1}^*)^\top$, there is an action $(p^1, p^1, q^1) \in \mathcal{A}^1$ with $(p_1^1, \ldots, p_{N-1}^1)^\top \in O$ such that (p^1, p^1, q^1) is a better response against (p^*, p^*, q^*) than the action (p^*, p^*, q^*) itself, p^1 not being a Walrasian equilibrium price system. This means that, generically, proposing price regulations which exclude the Walrasian equilibrium price system is a better response against a certain Walrasian equilibrium than proposing this Walrasian equilibrium.

Consider the political economic system $\overline{\mathcal{E}}$ with status quo a Walrasian equilibrium (p^*, x^*) of the economy $\mathcal{E}$ with $p_N^* = 1$. Suppose political candidate 2 chooses the action $0^{N-1} \in \widetilde{\mathcal{A}}^2$, i.e., political candidate 2 proposes to stay at the status quo. Then it will be shown that only in degenerate cases the best response of political candidate 1 is also choosing action 0^{N-1}. Even stronger, it will be shown that, generically, $\overline{K}^1 \neq 0^{N-1}$. Using symmetry considerations it follows that, generically, $\overline{K}^2 \neq 0^{N-1}$. So from Theorem 4.1 it follows that in a directional political economic equilibrium, generically, both political candidates choose to move away from the status quo.

Assumptions which have to be made are that there is at least one non-numeraire commodity, as is always assumed in this paper, and that there are at least two consumers. Otherwise, every consumer will keep his initial endowments in every Walrasian or Drèze equilibrium. Hence, it is impossible to influence the voting decision of consumers by proposing price regulations. The set of all possible voting functions of a consumer $i \in I_M$ for political candidate 1 satisfying Assumption A7 is denoted by Π^i. Hence,

$$\begin{aligned}\Pi^i = \{\pi^{i1} \in C^2((0,1)\times(0,1),(0,1)) |\, & \partial_{v^1}\pi^{i1}(\overline{v}^1,\overline{v}^2) > 0,\ \forall(\overline{v}^1,\overline{v}^2) \in (0,1)\times(0,1) \\ & \partial_{v^2}\pi^{i1}(\overline{v}^1,\overline{v}^2) < 0,\ \forall(\overline{v}^1,\overline{v}^2) \in (0,1)\times(0,1)\}.\end{aligned}$$

This set is given the topology induced by the C^1-topology. Notice that Π^i, $\forall i \in I_M$, is open in $C^2((0,1)\times(0,1),\mathbb{R})$. Define the set Π by $\Pi = \prod_{i\in I_M}\Pi^i$ and give this set the product topology. Notice that, for every $\pi \in \Pi$, $\pi = (\pi^{11}, \ldots, \pi^{M1})$ with π^{i1} denoting the voting function of consumer $i \in I_M$. For some consumer $i \in I_M$, let $\pi^{i1} \in \Pi^i$ be given and let

$\epsilon \in C^0((0,1) \times (0,1), \mathbf{R}_{++})$ be given. Define the set $V_{\pi^{i1},\epsilon}$ by

$$\begin{aligned} V_{\pi^{i1},\epsilon} = \{ & f \in C^2((0,1) \times (0,1), (0,1)) \mid \\ & |\pi^{i1}(\overline{v}^1, \overline{v}^2) - f(\overline{v}^1, \overline{v}^2)| < \epsilon(\overline{v}^1, \overline{v}^2), \quad \forall (\overline{v}^1, \overline{v}^2) \in (0,1) \times (0,1), \\ & |\partial_{v^1} \pi^{i1}(\overline{v}^1, \overline{v}^2) - \partial_{v^1} f(\overline{v}^1, \overline{v}^2)| < \epsilon(\overline{v}^1, \overline{v}^2), \ \forall (\overline{v}^1, \overline{v}^2) \in (0,1) \times (0,1), \\ & |\partial_{v^2} \pi^{i1}(\overline{v}^1, \overline{v}^2) - \partial_{v^2} f(\overline{v}^1, \overline{v}^2)| < \epsilon(\overline{v}^1, \overline{v}^2), \ \forall (\overline{v}^1, \overline{v}^2) \in (0,1) \times (0,1)\}. \end{aligned}$$

Notice that $V_{\pi^{i1},\epsilon}$ is a member of the base for the C^1-topology on $C^2((0,1) \times (0,1), (0,1))$.

Let the political economic system $\hat{\mathcal{E}}$ be given. Let (p^*, x^*) with $p^*_N = 1$ and $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$, $\forall j \in I_{N-1}$, be a locally unique, regular Walrasian equilibrium of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$. Let $q^* \in Q^{N-1}$ be such that the proposal (p^*, p^*, q^*) corresponds to this Walrasian equilibrium. Suppose political candidate 2 proposes (p^*, p^*, q^*). Then it holds that (p^*, p^*, q^*) is a best response for political candidate 1 if and only if $K^1((p^*, p^*, q^*), (p^*, p^*, q^*)) = \max_{(a^1, q^1) \in \mathcal{A}^1} K^1((a^1, q^1), (p^*, p^*, q^*))$. From Theorem 3.2 it follows that a necessary condition for (p^*, p^*, q^*) to be a best response of political candidate 1 is given by

$$\begin{aligned} \sum_{i \in I_M} \Big(& \partial_{v^1} \pi^{i1} \left(\hat{v}^i(p^*_1, \ldots, p^*_{N-1}), \hat{v}^i(p^*_1, \ldots, p^*_{N-1}) \right) \partial_{(p_1, \ldots, p_{N-1})^\top} \hat{v}^i(p^*_1, \ldots, p^*_{N-1}) \\ & - \partial_{v^1} \pi^{i2} \left(\hat{v}^i(p^*_1, \ldots, p^*_{N-1}), \hat{v}^i(p^*_1, \ldots, p^*_{N-1}) \right) \partial_{(p_1, \ldots, p_{N-1})^\top} \hat{v}^i(p^*_1, \ldots, p^*_{N-1}) \Big) = 0^{N-1^\top}. \end{aligned} \tag{12}$$

From Theorem 4.1 and the remark made below it, it follows that (12) is a necessary and sufficient condition for 0^{N-1} to be a possible action of political candidate 1 in a directional political economic equilibrium of the political economic system $\overline{\mathcal{E}}$ with status quo (p^*, x^*). Using that $\pi^{i1}(v^1, v^2) + \pi^{i2}(v^1, v^2) = 1$, $\forall (v^1, v^2) \in (0,1) \times (0,1)$ and using Theorem 3.2, it follows that (12) is equivalent to

$$- \sum_{i \in I_M} \partial_{v^1} \pi^{i1} \left(u^i(x^{*i}), u^i(x^{*i}) \right) \partial_{x_N} u^i(x^{*i})(x^{*i}_j - \omega^i_j) = 0, \forall j \in I_{N-1}. \tag{13}$$

Notice that the expression in (13) does not depend on π^{i2}, $\forall i \in I_M$. For the remainder of this section, the voting function π^{i1}, $\forall i \in I_M$, will therefore be denoted by π^i.

Let $(X^i, u^i)_{i \in I_M}$ satisfy the Assumptions A1-A2. Let the set Ω^1 be as in Theorem 3.4. For every $\omega \in \Omega^1$, let $k(\omega)$ be the number of Walrasian equilibria in the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$. By Theorem 3.4, for every $\overline{\omega} \in \Omega^1$, there exists an open set O containing $\overline{\omega}$ and being such that, for every $\omega \in O$, $k(\omega) = k(\overline{\omega})$ and the vector of Walrasian equilibria $((p(\omega, 1), x(\omega, 1)), \ldots, (p(\omega, k(\overline{\omega})), x(\omega, k(\overline{\omega}))))$ depends in a continuous way on ω. Define the set $\mathcal{W}^1$ by

$$\begin{aligned} \mathcal{W}^1 = \{ & (\omega, \pi) \in \Omega^1 \times \Pi \mid \forall k \in I_{k(\omega)}, \ \exists i^1, i^2 \in I_M, \\ & \partial_{v^1} \pi^{i^1} \left(u^{i^1}(x^{i^1}(\omega, k)), u^{i^1}(x^{i^1}(\omega, k)) \right) \partial_{x_N} u^{i^1}(x^{i^1}(\omega, k)) \\ & \neq \partial_{v^1} \pi^{i^2} \left(u^{i^2}(x^{i^2}(\omega, k)), u^{i^2}(x^{i^2}(\omega, k)) \right) \partial_{x_N} u^{i^2}(x^{i^2}(\omega, k)) \Big\}. \end{aligned} \tag{14}$$

In Theorem 4.2 it is shown that $\mathcal{W}^1$ is open in $\Omega \times \Pi$ and in Theorem 4.3 that $\mathcal{W}^1$ is dense in $\Omega \times \Pi$.

Theorem 4.2

Let $(X^i, u^i)_{i \in I_M}$ satisfy the Assumptions A1-A2 and let $M \in \mathbb{N} \setminus \{1\}$. Then the set $\mathcal{W}^1$ is open in $\Omega \times \Pi$.

Proof

Let some $i \in I_M$ be given. Let the function $g^i : \Pi^i \times (0,1) \times (0,1) \to \mathbb{R}_{++}$ be defined by

$$g^i(\overline{\pi}^i, \overline{v}^1, \overline{v}^2) = \partial_{v^1} \overline{\pi}^i(\overline{v}^1, \overline{v}^2), \; \forall (\overline{\pi}^i, \overline{v}^1, \overline{v}^2) \in \Pi^i \times (0,1) \times (0,1).$$

It will be shown that g^i is continuous. Let O be an open set in $\mathbb{R}_{++}$ and let $(\widehat{\pi}^i, \widehat{v}^1, \widehat{v}^2) \in {g^i}^{-1}(O)$ be given. Let $\varepsilon \in \mathbb{R}_{++}$ be such that $B^1(g^i(\widehat{\pi}^i, \widehat{v}^1, \widehat{v}^2), \varepsilon) \subset O$. Let the function $\epsilon : (0,1) \times (0,1) \to \mathbb{R}_{++}$ be defined by

$$\epsilon(v^1, v^2) = \tfrac{1}{2}\varepsilon, \; \forall (v^1, v^2) \in (0,1) \times (0,1).$$

Since $\partial_{v^1} \widehat{\pi}^i : (0,1) \times (0,1) \to \mathbb{R}_{++}$ and $\partial_{v^2} \widehat{\pi}^i : (0,1) \times (0,1) \to -\mathbb{R}_{++}$ are continuous functions, there exists $\delta \in \mathbb{R}_{++}$ such that $(\overline{v}^1, \overline{v}^2) \in B^2((\widehat{v}^1, \widehat{v}^2)^\top, \delta)$ implies

$$\|(\partial_{(v^1,v^2)^\top} \widehat{\pi}^i(\overline{v}^1, \overline{v}^2) - \partial_{(v^1,v^2)^\top} \widehat{\pi}^i(\widehat{v}^1, \widehat{v}^2))^\top\|_2 < \tfrac{1}{2}\varepsilon.$$

For every $(\overline{\pi}^i, \overline{v}^1, \overline{v}^2) \in V_{\widehat{\pi}^i, \epsilon} \times B^2((\widehat{v}^1, \widehat{v}^2)^\top, \delta)$, it holds that

$$\begin{aligned}
&|\partial_{v^1} \overline{\pi}^i(\overline{v}^1, \overline{v}^2) - \partial_{v^1} \widehat{\pi}^i(\widehat{v}^1, \widehat{v}^2)| \\
&\quad \leq |\partial_{v^1} \overline{\pi}^i(\overline{v}^1, \overline{v}^2) - \partial_{v^1} \widehat{\pi}^i(\overline{v}^1, \overline{v}^2)| + |\partial_{v^1} \widehat{\pi}^i(\overline{v}^1, \overline{v}^2) - \partial_{v^1} \widehat{\pi}^i(\widehat{v}^1, \widehat{v}^2)| \\
&\quad < \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon = \varepsilon,
\end{aligned}$$

so $g^i(\overline{\pi}^i, \overline{v}^1, \overline{v}^2) \in B^1(g^i(\widehat{\pi}^i, \widehat{v}^1, \widehat{v}^2), \varepsilon) \subset O$. Hence, ${g^i}^{-1}(O)$ is open, concluding the proof that g^i is a continuous function.

Let some $\overline{\omega} \in \Omega^1$ be given. Let O be a set open in Ω^1, containing $\overline{\omega}$, and being such that $k(\omega) = k(\overline{\omega})$, $\forall \omega \in O$, while, for every $k \in I_{k(\overline{\omega})}$, the function $f^{k,i} : O \to \mathbb{R}^N$, defined by

$$f^{k,i}(\omega) = x^i(\omega, k), \; \forall \omega \in O,$$

is continuous. Let the function $h : O \times \Pi \to \mathbb{R}^{k(\overline{\omega})M}$ be defined by associating with every $(\omega, \pi) \in O \times \Pi$ the element

$$\begin{aligned}
h(\omega, \pi) \;=\; &\Big(g^1\left(\pi^1, u^1(f^{1,1}(\omega)), u^1(f^{1,1}(\omega))\right) \partial_{x_N} u^1(f^{1,1}(\omega)), \ldots, \\
&g^M\left(\pi^M, u^M(f^{k(\overline{\omega}),M}(\omega)), u^M(f^{k(\overline{\omega}),M}(\omega))\right) \partial_{x_N} u^M(f^{k(\overline{\omega}),M}(\omega))\Big)^\top.
\end{aligned}$$

Using the continuity of the functions $f^{k,i}$, $\forall k \in I_{k(\overline{\omega})}$, $\forall i \in I_M$, and g^i, $\forall i \in I_M$, and the fact that u^i is continuously differentiable, it follows easily that h is a continuous function. Let the open set $T_{\overline{\omega}}$ be defined by

$$T_{\overline{\omega}} = \left\{ t \in \mathbb{R}^{k(\overline{\omega})M} \,\middle|\, \forall k \in I_{k(\overline{\omega})},\ \exists i^1, i^2 \in I_M,\ t_k^{i^1} \neq t_k^{i^2} \right\}.$$

Clearly, the set $\mathcal{W}_{\overline{\omega}}$, defined by $\mathcal{W}_{\overline{\omega}} = h^{-1}(T_{\overline{\omega}})$, is open in $O \times \Pi$ and therefore open in $\Omega \times \Pi$. So, $\mathcal{W}^1 = \cup_{\overline{\omega} \in \Omega^1} \mathcal{W}_{\overline{\omega}}$ is open in $\Omega \times \Pi$. Q.E.D.

Theorem 4.3

Let $(X^i, u^i)_{i \in I_M}$ satisfy the Assumptions A1-A2 and let $M \in \mathbb{N} \setminus \{1\}$. Then the set $\mathcal{W}^1$ is dense in $\Omega \times \Pi$.

Proof

It will be shown that the closure of $\mathcal{W}^1$ in $\Omega \times \Pi$ contains $\Omega^1 \times \Pi$, a set being dense in $\Omega \times \Pi$, thereby showing the result. Let an element $\overline{\omega} \in \Omega^1$ and a set $\mathcal{O}$ being open in Π be given. Let $\overline{\pi}^i \in \Pi^i$, $\forall i \in I_M$, and $\epsilon^i \in C^0((0,1) \times (0,1), \mathbb{R}_{++})$, $\forall i \in I_M$, be such that $\prod_{i \in I_M} V_{\overline{\pi}^i, \epsilon^i} \subset \mathcal{O}$. It will be shown that $\mathcal{W}^1 \cap (\{\overline{\omega}\} \times \prod_{i \in I_M} V_{\overline{\pi}^i, \epsilon^i}) \neq \emptyset$, thereby showing that the closure of $\mathcal{W}^1$ in $\Omega \times \Pi$ contains $\Omega^1 \times \Pi$.

For every $k \in I_{k(\overline{\omega})}$, for every $i \in I_M$, let the real numbers $\overline{v}^{k,i}$ and $\overline{d}^{k,i}$ be defined by $\overline{v}^{k,i} = u^i(x^i(\overline{\omega}, k))$ and $\overline{d}^{k,i} = \partial_{x_N} u^i(x^i(\overline{\omega}, k))$. Let the set K^1 be defined by

$$K^1 = \left\{ k \in I_{k(\overline{\omega})} \,\middle|\, \partial_{v^1} \overline{\pi}^1(\overline{v}^{k,1}, \overline{v}^{k,1}) \overline{d}^{k,1} = \partial_{v^1} \overline{\pi}^i(\overline{v}^{k,i}, \overline{v}^{k,i}) \overline{d}^{k,i},\ \forall i \in I_M \right\}.$$

If $K^1 = \emptyset$, then the proof is finished. Consider the case where $K^1 \neq \emptyset$. Let K^2 be a maximal subset of K^1 such that $k^1, k^2 \in K^2$ and $k^1 \neq k^2$ implies $\overline{v}^{k^1,1} \neq \overline{v}^{k^2,1}$. So, $\{\overline{v}^{k,1} \mid k \in K^1\} = \{\overline{v}^{k,1} \mid k \in K^2\}$. Let the, possibly empty, set K^3 be defined by

$$K^3 = \left\{ k \in I_{k(\overline{\omega})} \setminus K^1 \,\middle|\, \partial_{v^1} \overline{\pi}^1(\overline{v}^{k,1}, \overline{v}^{k,1}) \overline{d}^{k,1} \neq \partial_{v^1} \overline{\pi}^2(\overline{v}^{k,2}, \overline{v}^{k,2}) \overline{d}^{k,2} \right\}.$$

For every $\widehat{v} \in (0,1)$, for every $\delta \in \mathbb{R}_{++}$ satisfying $\overline{B}^2((\widehat{v}, \widehat{v})^\top, \delta) \subset (0,1) \times (0,1)$, for every $\varepsilon \in \mathbb{R}_{++}$, let the function $f_{\widehat{v},\delta,\varepsilon} \in C^\infty((0,1) \times (0,1), \mathbb{R}_+)$ have the following properties:

1. $f_{\widehat{v},\delta,\varepsilon}(\overline{v}^1, \overline{v}^2) = 0$, $\forall (\overline{v}^1, \overline{v}^2) \in ((0,1) \times (0,1)) \setminus B^2((\widehat{v}, \widehat{v})^\top, \delta)$,

2. $\|\partial_{(v^1,v^2)^\top} f_{\widehat{v},\delta,\varepsilon}(\overline{v}^1, \overline{v}^2)\|_2 < \varepsilon$, $\forall (\overline{v}^1, \overline{v}^2) \in (0,1) \times (0,1)$,

3. $\partial_{v^1} f_{\widehat{v},\delta,\varepsilon}(\widehat{v} - \frac{1}{2}\delta, \widehat{v} - \frac{1}{2}\delta) > 0$.

It is not difficult to show that such a function $f_{\widehat{v},\delta,\varepsilon}$ exists, see for instance Hirsch (1976), page 41-42. Let the function $\widehat{\pi} \in \Pi$ be defined by

$$\widehat{\pi}^1(v^1, v^2) = \overline{\pi}^1(v^1, v^2) + \sum_{k \in K^2} f_{\widehat{v}^k, \delta^k, \varepsilon^k}(v^1, v^2),\ \forall (v^1, v^2) \in (0,1) \times (0,1),$$

and $\widehat{\pi}^i = \overline{\pi}^i$, $\forall i \in I_M \setminus \{1\}$, where, for every $k \in K^2$,

$$
\begin{aligned}
\delta^k &= \min\left(\left\{\tfrac{1}{2}|\overline{v}^{k,1} - \overline{v}^{k',1}| \,\middle|\, k' \in I_{k(\overline{\omega})} \text{ and } \overline{v}^{k',1} \neq \overline{v}^{k,1}\right\} \cup \left\{\tfrac{1}{2}\overline{v}^{k,1}, \tfrac{1}{2}(1-\overline{v}^{k,1})\right\}\right), \\
\widehat{v}^k &= \overline{v}^{k,1} + \tfrac{1}{2}\delta^k, \\
\varepsilon^k &= \min\left(\left\{\partial_{v^1}\overline{\pi}^1(\overline{v}^1, \overline{v}^2) \,\middle|\, (\overline{v}^1, \overline{v}^2) \in \overline{B}^2((\widehat{v}^k, \widehat{v}^k)^\top, \delta^k)\right\}\right. \\
&\qquad \cup \left\{-\partial_{v^2}\overline{\pi}^1(\overline{v}^1, \overline{v}^2) \,\middle|\, (\overline{v}^1, \overline{v}^2) \in \overline{B}^2((\widehat{v}^k, \widehat{v}^k)^\top, \delta^k)\right\} \\
&\qquad \cup \left\{\epsilon^1(\overline{v}^1, \overline{v}^2) \,\middle|\, (\overline{v}^1, \overline{v}^2) \in \overline{B}^2((\widehat{v}^k, \widehat{v}^k)^\top, \delta^k)\right\} \\
&\qquad \left.\cup \left\{|\partial_{v^1}\overline{\pi}^1(\overline{v}^{k,1}, \overline{v}^{k,1})\overline{d}^{k,1} - \partial_{v^1}\overline{\pi}^2(\overline{v}^{k,2}, \overline{v}^{k,2})\overline{d}^{k,2}| \,\middle|\, k \in K^3\right\}\right).
\end{aligned}
$$

For every $k \in K^2$, the choice of δ^k guarantees that $\widehat{\pi}$ is a perturbation of $\overline{\pi}$ on non-intersecting neighbourhoods of utilities consumer 1 derives at Walrasian equilibria of the economy $\mathcal{E} = ((X^i, u^i, \overline{\omega}^i)_{i \in I_M})$. For every $k \in K^2$, the choice of $\widehat{v}^k$ guarantees that the voting behaviour of consumer 1 is indeed perturbed at the utility he derives at Walrasian equilibrium k. Finally, ε^k, $\forall k \in K^2$, is chosen such that $\widehat{\pi} \in \prod_{i \in I_M} V_{\overline{\pi}^i, \epsilon^i}$ and the perturbation is so small that, for every $k \in K^3$, the already existing inequality of $\partial_{v^1}\overline{\pi}^1(\overline{v}^{k,1}, \overline{v}^{k,1})\overline{d}^{k,1}$ and $\partial_{v^1}\overline{\pi}^2(\overline{v}^{k,1}, \overline{v}^{k,1})\overline{d}^{k,2}$ remains. The fact that $\overline{d}^{k,i} > 0$, $\forall k \in I_{k(\overline{\omega})}$, $\forall i \in I_M$, guarantees that

$$\partial_{v^1}\widehat{\pi}^1(\overline{v}^{k,1}, \overline{v}^{k,1})\overline{d}^{k,1} \neq \partial_{v^1}\widehat{\pi}^2(\overline{v}^{k,2}, \overline{v}^{k,2})\overline{d}^{k,2}, \ \forall k \in K^1.$$

Clearly,

$$\partial_{v^1}\widehat{\pi}^1(\overline{v}^{k,1}, \overline{v}^{k,1})\overline{d}^{k,1} \neq \partial_{v^1}\widehat{\pi}^2(\overline{v}^{k,2}, \overline{v}^{k,2})\overline{d}^{k,2}, \ \forall k \in K^3.$$

Let $k' \in I_{k(\overline{\omega})} \setminus (K^1 \cup K^3)$ be given. Then $\partial_{v^1}\overline{\pi}^1(\overline{v}^{k',1}, \overline{v}^{k',1})\overline{d}^{k',1} = \partial_{v^1}\overline{\pi}^2(\overline{v}^{k',2}, \overline{v}^{k',2})\overline{d}^{k',2}$, while there exists $i' \in I_M$ such that $\partial_{v^1}\overline{\pi}^1(\overline{v}^{k',1}, \overline{v}^{k',1})\overline{d}^{k',1} \neq \partial_{v^1}\overline{\pi}^{i'}(\overline{v}^{k',i'}, \overline{v}^{k',i'})\overline{d}^{k',i'}$. If $\overline{v}^{k',1} \in \{\overline{v}^{k,1} \mid k \in K^2\}$, then, since $\overline{d}^{k',1} > 0$,

$$\partial_{v^1}\widehat{\pi}^1(\overline{v}^{k',1}, \overline{v}^{k',1})\overline{d}^{k',1} \neq \partial_{v^1}\widehat{\pi}^2(\overline{v}^{k',2}, \overline{v}^{k',2})\overline{d}^{k',2}.$$

If $\overline{v}^{k',1} \notin \{\overline{v}^{k,1} \mid k \in K^2\}$, then the choice of δ^k, $\forall k \in K^2$, guarantees that

$$
\begin{aligned}
\partial_{v^1}\widehat{\pi}^1(\overline{v}^{k',1}, \overline{v}^{k',1})\overline{d}^{k',1} &= \partial_{v^1}\overline{\pi}^1(\overline{v}^{k',1}, \overline{v}^{k',1})\overline{d}^{k',1} \\
&\neq \partial_{v^1}\overline{\pi}^{i'}(\overline{v}^{k',i'}, \overline{v}^{k',i'})\overline{d}^{k',i'} = \partial_{v^1}\widehat{\pi}^{i'}(\overline{v}^{k',i'}, \overline{v}^{k',i'})\overline{d}^{k',i'}.
\end{aligned}
$$

So, it follows that $(\overline{\omega}, \widehat{\pi}) \in \mathcal{W}^1$, whereas, clearly, $(\overline{\omega}, \widehat{\pi}) \in \{\overline{\omega}\} \times \prod_{i \in I_M} V_{\overline{\pi}^i, \epsilon^i}$. Q.E.D.

Theorem 4.4

Let $(X^i, u^i)_{i \in I_M}$ satisfy the Assumptions A1-A2 and let $M \in \mathbf{N} \setminus \{1\}$. Then there exists an open and dense set $\mathcal{W}^2$ in $\Omega \times \Pi$ such that, for every $(\omega, \pi) \in \mathcal{W}^2$, for every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ it holds that $-\sum_{i \in I_M} \partial_{v^1}\pi^i(u^i(x^{*i}), u^i(x^{*i}))\partial_{x_N}u^i(x^{*i})(x^{*i}_j - \omega^i_j) \neq 0$ for some $j \in I_{N-1}$.

Proof

By Theorem 4.2 and Theorem 4.3 the set $\mathcal{W}^1$ is open and dense in $\Omega \times \Pi$. Let $\overline{\pi} \in \Pi$ be such that the set $\Omega_{\overline{\pi}}$, defined by

$$\Omega_{\overline{\pi}} = \left\{ \omega \in \Omega \,\middle|\, (\omega, \overline{\pi}) \in \mathcal{W}^1 \right\},$$

is non-empty. Since $\mathcal{W}^1$ is open in $\Omega \times \Pi$, it holds that $\Omega_{\overline{\pi}}$ is open in Ω. Let $\overline{\omega} \in \Omega_{\overline{\pi}}$ and $(p', \overline{x}) \in \mathbb{R}^N_{++} \times X$ with $p'_N = 1$ be given. If $(p', \overline{x})$ is a Walrasian equilibrium of the economy $\mathcal{E} = ((X^i, u^i, \overline{\omega}^i)_{i \in I_M})$ satisfying $-\sum_{i \in I_M} \partial_{v^1} \overline{\pi}^i(u^i(\overline{x}^i), u^i(\overline{x}^i)) \partial_{x_N} u^i(\overline{x}^i)(\overline{x}^i_j - \overline{\omega}^i_j) = 0$, $\forall j \in I_{N-1}$, then there exists $\overline{\lambda}^i \in \mathbb{R}$, $\forall i \in I_M$, such that

$$\begin{aligned}
\partial_{x^i} u^i(\overline{x}^i)^\top - \overline{\lambda}^i (p'_1, \ldots, p'_{N-1}, 1)^\top &= 0^N, \ \forall i \in I_M, & (15)\\
(p'_1, \ldots, p'_{N-1}, 1)\overline{x}^i - (p'_1, \ldots, p'_{N-1}, 1)\overline{\omega}^i &= 0, \quad \forall i \in I_M, & (16)\\
\textstyle\sum_{i \in I_M} \overline{x}^i_j - \sum_{i \in I_M} \overline{\omega}^i_j &= 0, \quad \forall j \in I_{N-1}, & (17)\\
-\textstyle\sum_{i \in I_M} \partial_{v^1} \overline{\pi}^i \left(u^i(\overline{x}^i), u^i(\overline{x}^i)\right) \partial_{x_N} u^i(\overline{x}^i)(\overline{x}^i_1 - \overline{\omega}^i_1) &= 0. & (18)
\end{aligned}$$

Notice that the condition that on the market of the numeraire commodity the total excess demand is equal to zero is not specified. This condition is implied by the equations in (16) and (17). Let the function

$$\psi : X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times \Omega_{\overline{\pi}} \to \mathbb{R}^{MN+M+N}$$

be defined such that $\psi(x, \lambda, (p_1, \ldots, p_{N-1})^\top, \omega)$ is the left-hand side of (15)-(18), for every $(x, \lambda, (p_1, \ldots, p_{N-1})^\top, \omega) \in X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times \Omega_{\overline{\pi}}$. For every $\omega \in \Omega$, the function

$$\psi^\omega : X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \to \mathbb{R}^{MN+M+N}$$

is defined by associating with every $(x, \lambda, (p_1, \ldots, p_{N-1})^\top) \in X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++}$ the element $\psi(x, \lambda, (p_1, \ldots, p_{N-1})^\top, \omega)$. The matrix of partial derivatives of ψ evaluated at a point $\overline{\xi} = (\overline{x}, \overline{\lambda}, (p'_1, \ldots, p'_{N-1})^\top, \overline{\omega}) \in X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++} \times \Omega_{\overline{\pi}}$ such that $\psi(\overline{\xi}) = 0^{MN+M+N}$ is denoted by $\overline{M}$ and is given in Table 4.1. It will be shown that the matrix $\overline{M}$ has rank $MN + M + N$. Notice that

$$\begin{aligned}
\partial_{\omega^i_1} \psi_{MN+M+N}(\overline{\xi}) &= \partial_{v^1} \overline{\pi}^i \left(u^i(\overline{x}^i), u^i(\overline{x}^i)\right) \partial_{x_N} u^i(\overline{x}^i), \forall i \in I_M, & (19)\\
\partial_{\omega^i_j} \psi_{MN+M+N}(\overline{\xi}) &= 0, \qquad \forall i \in I_M, \forall j \in I_N \setminus \{1\}. & (20)
\end{aligned}$$

The partial derivatives of ψ_{MN+M+N} at $\overline{\xi}$ with respect to x are quite complicated, but do not matter in the following.

Let $y \in \mathbb{R}^{MN+M+N}$ be such that $y^\top \overline{M} = 0^{2MN+M+N-1^\top}$. Then, $y^\top \partial_{\omega^i_N} \psi(\overline{\xi}) = 0$, $\forall i \in I_M$, implies, using (20), that

$$y_{MN+i} = 0, \ \forall i \in I_M. \qquad (21)$$

$$
\begin{array}{cccc|c}
\begin{array}{ccc} \partial^2_{x^1x^1}u^1(\overline{x}^1) & & 0 \\ & \ddots & \\ 0 & & \partial^2_{x^Mx^M}u^M(\overline{x}^M) \end{array} &
\begin{array}{ccc} \begin{array}{c} -p'_1 \\ \vdots \\ -p'_{N-1} \\ -1 \end{array} & & 0 \\ & \ddots & \\ 0 & & \begin{array}{c} -p'_1 \\ \vdots \\ -p'_{N-1} \\ -1 \end{array} \end{array} &
\begin{array}{c} \begin{array}{c} -\overline{\lambda}^1 I^{N-1} \\ 0^{N-1^\top} \end{array} \\ \vdots \\ \begin{array}{c} -\overline{\lambda}^M I^{N-1} \\ 0^{N-1^\top} \end{array} \end{array} &
0^{MN\times MN} & MN \\
\begin{array}{ccc} (p'_j)_{j\in I_{N-1}}\ 1 & & 0 \\ & \ddots & \\ 0 & & (p'_j)_{j\in I_{N-1}}\ 1 \end{array} &
0^{M\times M} &
\begin{array}{c} (\overline{x}^1_j-\overline{\omega}^1_j)_{j\in I_{N-1}} \\ \vdots \\ (\overline{x}^M_j-\overline{\omega}^M_j)_{j\in I_{N-1}} \end{array} &
\begin{array}{ccc} (-p'_j)_{j\in I_{N-1}}\ -1 & & 0 \\ & \ddots & \\ 0 & & (-p'_j)_{j\in I_{N-1}}\ -1 \end{array} & M \\
I^{N-1}\ 0^{N-1}\ \cdots\ I^{N-1}\ 0^{N-1} & 0^{(N-1)\times M} & 0^{(N-1)\times(N-1)} & -I^{N-1}\ 0^{N-1}\ \cdots\ -I^{N-1}\ 0^{N-1} & N-1 \\
\partial_x\psi_{MN+M+N}(\overline{\xi}) & 0^{M^\top} & 0^{N-1^\top} & \partial_\omega\psi_{MN+M+N}(\overline{\xi}) & 1 \\
\hline
MN & M & N-1 & MN &
\end{array}
$$

Table 4.1. The matrix $\overline{M}$.

By the definition of $\Omega_{\overline{\pi}}$ there exists $i^1, i^2 \in I_M$ such that

$$\partial_{v^1}\overline{\pi}^{i^1}\left(u^{i^1}(\overline{x}^{i^1}), u^{i^1}(\overline{x}^{i^1})\right)\partial_{x_N}u^{i^1}(\overline{x}^{i^1}) \neq \partial_{v^1}\overline{\pi}^{i^2}\left(u^{i^2}(\overline{x}^{i^2}), u^{i^2}(\overline{x}^{i^2})\right)\partial_{x_N}u^{i^2}(\overline{x}^{i^2}). \tag{22}$$

Moreover, (19), (21), and $y^\top \partial_{\omega^i_1}\psi(\overline{\xi}) = 0$, $\forall i \in \{i^1, i^2\}$, implies

$$-y_{MN+M+1} + \partial_{v^1}\overline{\pi}^{i^1}\left(u^{i^1}(\overline{x}^{i^1}), u^{i^1}(\overline{x}^{i^1})\right)\partial_{x_N}u^{i^1}(\overline{x}^{i^1})y_{MN+M+N} = 0, \tag{23}$$

$$-y_{MN+M+1} + \partial_{v^1}\overline{\pi}^{i^2}\left(u^{i^2}(\overline{x}^{i^2}), u^{i^2}(\overline{x}^{i^2})\right)\partial_{x_N}u^{i^2}(\overline{x}^{i^2})y_{MN+M+N} = 0. \tag{24}$$

So,

$$\begin{aligned}\Big(\partial_{v^1}\overline{\pi}^{i^1}\left(u^{i^1}(\overline{x}^{i^1}), u^{i^1}(\overline{x}^{i^1})\right)\partial_{x_N}u^{i^1}(\overline{x}^{i^1})& \\ -\partial_{v^1}\overline{\pi}^{i^2}\left(u^{i^2}(\overline{x}^{i^2}), u^{i^2}(\overline{x}^{i^2})\right)&\partial_{x_N}u^{i^2}(\overline{x}^{i^2})\Big)y_{MN+M+N} = 0,\end{aligned}$$

implying by (22) that $y_{MN+M+N} = 0$. Now it can be shown similarly as in the proof of Theorem 3.6 that $y = 0^{MN+M+N}$, so $\overline{M}$ has rank $MN + M + N$, and ψ intersects $\{0^{MN+M+N}\}$ transversally, $\psi \pitchfork \{0^{MN+M+N}\}$. Let the set $\overline{\Omega}_{\overline{\pi}}$ be defined by

$$\overline{\Omega}_{\overline{\pi}} = \left\{\omega \in \Omega_{\overline{\pi}} \,\middle|\, \psi^\omega \pitchfork \{0^{MN+M+N}\}\right\}.$$

From the transversality theorem it follows that the set $\Omega_{\overline{\pi}} \setminus \overline{\Omega}_{\overline{\pi}}$ has Lebesgue measure zero. For every $\omega \in \overline{\Omega}_{\overline{\pi}}$, ψ^{ω} is a function from an $(MN + M + N - 1)$-dimensional C^{∞} manifold into an $(MN + M + N)$-dimensional C^{∞} manifold, $\{0^{MN+M+N}\}$ is a 0-dimensional C^{∞} manifold, $\psi^{\omega} \in C^1(X \times \mathbb{R}^M \times \mathbb{R}^{N-1}_{++}, \mathbb{R}^{MN+M+N})$, and $\psi^{\omega} \,\overline{\pitchfork}\, \{0^{MN+M+N}\}$, so it follows that $\psi^{\omega^{-1}}(\{0^{MN+M+N}\}) = \emptyset$. Let the set $\overline{\mathcal{W}}$ be defined by

$$\overline{\mathcal{W}} = \left\{(\omega, \pi) \in \mathcal{W}^1 \,\middle|\, \omega \in \overline{\Omega}_{\pi} \right\}.$$

Clearly, the closure of $\overline{\mathcal{W}}$ in $\mathcal{W}^1$ is equal to $\mathcal{W}^1$. Since $\mathcal{W}^1$ is dense in $\Omega \times \Pi$ it holds that $\overline{\mathcal{W}}$ is dense in $\Omega \times \Pi$. Let the set $\mathcal{W}^2$ be defined by the elements $(\omega, \pi) \in \Omega^1 \times \Pi$ such that every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ satisfies $-\sum_{i \in I_M} \partial_{v^1} \pi^i(u^i(x^{*i}), u^i(x^{*i}))\partial_{x_N} u^i(x^{*i})(x^{*i}_j - \omega^i_j) \neq 0$ for some $j \in I_{N-1}$. Clearly, $\overline{\mathcal{W}} \subset \mathcal{W}^2$ and hence $\mathcal{W}^2$ is dense in $\Omega \times \Pi$. Similar to the case where it is shown that $\mathcal{W}^1$ is open in $\Omega \times \Pi$, it can be shown that $\mathcal{W}^2$ is open in $\Omega \times \Pi$. Q.E.D.

Combining the results of Theorem 3.7 and Theorem 4.4, the following result is obtained.

Theorem 4.5
Let $(X^i)_{i \in I_M}$ satisfy the Assumption A1 and let $M \in \mathbb{N}\setminus\{1\}$. Then there exists an open and dense set $\mathcal{V}$ in $U \times \Omega \times \Pi$ such that, for every $(u, \omega, \pi) \in \mathcal{V}$, every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ is locally unique, regular, for every $j \in I_{N-1}$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$, and

$$\sum_{i \in I_M} \partial_{v^1} \pi^i \left(\widehat{v}^i(p^*_1, \ldots, p^*_{N-1}), \widehat{v}^i(p^*_1, \ldots, p^*_{N-1})\right) \partial_{(p_1, \ldots, p_{N-1})^{\top}} \widehat{v}^i(p^*_1, \ldots, p^*_{N-1}) \neq 0^{N-1^{\top}}.$$

Proof
Let the set $\mathcal{V}^1$ be defined by $\mathcal{V}^1 = \mathcal{U}^3 \times \Pi$, where $\mathcal{U}^3$ is as in Theorem 3.7. The set $\mathcal{W}^2$ obtained in Theorem 4.4 depends on the choice of $u \in U$ and will therefore be denoted by $\mathcal{W}^2_u$, $\forall u \in U$. Let the set $\mathcal{V}^2$ be defined by

$$\mathcal{V}^2 = \left\{(u, \omega, \pi) \in U \times \Omega \times \Pi \,\middle|\, (\omega, \pi) \in \mathcal{W}^2_u \right\}.$$

Let the set $\mathcal{V}$ be defined as the set of elements $(u, \omega, \pi) \in \mathcal{V}^1$ such that, for every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$,

$$-\sum_{i \in I_M} \partial_{v^1} \pi^i \left(u^i(x^{*i}), u^i(x^{*i})\right) \partial_{x_N} u^i(x^{*i})(x^{*i}_j - \omega^i_j) \neq 0$$

for some $j \in I_{N-1}$. Notice that $\mathcal{V}^1 \cap \mathcal{V}^2 \subset \mathcal{V}$. Since $\mathcal{V} \subset \mathcal{V}^1$ it holds that, for every $(u, \omega, \pi) \in \mathcal{V}$, every Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ is locally unique, regular, and, for every $j \in I_{N-1}$, $\#\underline{I}_j(x^*) = \#\overline{I}_j(x^*) = 1$. It remains to be shown that $\mathcal{V}$ is open and dense in $U \times \Omega \times \Pi$. Since $\mathcal{U}^3$ is open and dense in $U \times \Omega$, it

holds that $\mathcal{V}^1 = \mathcal{U}^3 \times \Pi$ is open and dense in $U \times \Omega \times \Pi$. Clearly, $\mathcal{V}^2$ is dense in $U \times \Omega \times \Pi$. Since the intersection of an open and dense set with a dense set is dense, it holds that $\mathcal{V}^1 \cap \mathcal{V}^2$ is dense in $U \times \Omega \times \Pi$, so $\mathcal{V}$ is dense in $U \times \Omega \times \Pi$.
The Walrasian equilibrium set moves continuously in (u, ω), for every $(u, \omega) \in \mathcal{U}^3$, see Theorem 3.7. Since the voting functions have no influence on the Walrasian equilibrium set it clearly holds that the Walrasian equilibrium set moves continuously in (u, ω, π), for every $(u, \omega, \pi) \in \mathcal{V}^1$. In the proof of Theorem 4.2 it has been shown that, for every $i \in I_M$, the function $g^i : \Pi^i \times (0,1) \times (0,1) \to \mathbb{R}_{++}$, defined by $g^i(\overline{\pi}^i, \overline{v}^1, \overline{v}^2) = \partial_{v^1} \overline{\pi}^i(\overline{v}^1, \overline{v}^2)$, $\forall (\overline{\pi}^i, \overline{v}^1, \overline{v}^2) \in \Pi^i \times (0,1) \times (0,1)$, is continuous. Similarly, it can be shown that, for every $i \in I_M$, the function $f^i : U^i \times \mathbb{R}^N_{++} \to \mathbb{R}$, defined by $f^i(\overline{u}^i, \overline{x}^i) = \partial_{x_N} \overline{u}^i(\overline{x}^i)$, $\forall (\overline{u}^i, \overline{x}^i) \in U^i \times \mathbb{R}^N_{++}$, is continuous. Therefore, it follows easily that $\mathcal{V}$ is open in $\mathcal{V}^1$. Hence, $\mathcal{V}$ is open in $U \times \Omega \times \Pi$. Q.E.D.

Let the political economic system $\widehat{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_M}, (\widetilde{l}, \widetilde{L}), (A^k, (\pi^{ik})_{i \in I_M})_{k \in I_2})$ satisfy the Assumptions A1-A5 and A7, let $M \in \mathbb{N} \backslash \{1\}$, and let $(u, \omega, \pi) \in \mathcal{V}$ with $\mathcal{V}$ as in Theorem 4.5. Suppose political candidate 2 proposes the Walrasian equilibrium $(p^*, p^*, q^*) \in \mathcal{A}^2$. Using (12) and Theorem 4.5, it is clear that it is not optimal for political candidate 1 to choose the action $(p^*, p^*, q^*) \in \mathcal{A}^1$. Such a proposal can be improved by proposing $(p^1, p^1, q(p^1)) \in \mathcal{A}^1$, where p^1 can be chosen arbitrarily close to p^*. Since the Walrasian equilibria of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ are locally unique, p^1 can be chosen such that it is not a Walrasian equilibrium price system. By symmetry, there exists an open and dense set $\overline{\mathcal{V}}$ in $U \times \Omega \times \Pi$, for which the statements made above are true with the roles of the political candidates 1 and 2 reversed. Moreover, the set $\mathcal{V} \cap \overline{\mathcal{V}}$ is open and dense in $U \times \Omega \times \Pi$.

Let the political economic system $\overline{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_M}, (\widetilde{l}, \widetilde{L}), (\widetilde{\mathcal{A}}^k, (\pi^{ik})_{i \in I_M})_{k \in I_2})$ with status quo a Walrasian equilibrium (p^*, x^*) with $p^*_N = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$ satisfy the Assumptions A1-A4, A7, and A9, let $M \in \mathbb{N} \setminus \{1\}$, and let $(u, \omega, \pi) \in \mathcal{V}$ with $\mathcal{V}$ as in Theorem 4.5. It follows from Theorem 4.1 and Theorem 4.5 that political candidate 1 proposes $\overline{K}^1 / \|\overline{K}^1\|_2 \neq 0^{N-1}$ in a directional political economic equilibrium of the political economic system $\overline{\mathcal{E}}$ with status quo (p^*, x^*). By symmetry, there exists an open and dense set $\overline{\mathcal{V}}$ in $U \times \Omega \times \Pi$, for which the statements made above are true with the roles of the political candidates 1 and 2 reversed. Moreover, the set $\mathcal{V} \cap \overline{\mathcal{V}}$ is open and dense in $U \times \Omega \times \Pi$.

Corollary 4.6 and Corollary 4.7 are immediately obtained from Theorem 4.5 and the remarks being made in the previous two paragraphs.

Corollary 4.6
Let $(X^i)_{i \in I_M}, (\widetilde{l}, \widetilde{L}), (A^k)_{k \in I_2}$ satisfy the Assumptions A1 and A4-A5, and let $M \in \mathbb{N} \setminus \{1\}$. Then there exists an open and dense set of utility functions, initial endowments, and voting

functions $(u, \omega, \pi) \in U \times \Omega \times \Pi$ *such that there is no political economic equilibrium of the political economic system* $\widehat{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_M}, (\widetilde{l}, \widetilde{L}), (A^k, (\pi^{ik})_{i \in I_M})_{k \in I_2})$ *where both political candidates choose an action* $(p^*, p^*, q^*) \in \mathcal{A}^1 = \mathcal{A}^2$ *corresponding to a Walrasian equilibrium of the economy* $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$.

Corollary 4.7

Let $(X^i)_{i \in I_M}, (\widetilde{l}, \widetilde{L}), (\widetilde{\mathcal{A}}^k)_{k \in I_2}$ *satisfy the Assumptions A1, A4, and A9, and let* $M \in \mathbb{N} \setminus \{1\}$. *Then there exists an open and dense set of utility functions, initial endowments, and voting functions* $(u, \omega, \pi) \in U \times \Omega \times \Pi$ *such that in every directional political economic equilibrium* $(\widetilde{p}^{*1}, \widetilde{p}^{*2})$ *of the political economic system* $\overline{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_M}, (\widetilde{l}, \widetilde{L}), (\widetilde{\mathcal{A}}^k, (\pi^{ik})_{i \in I_M})_{k \in I_2})$ *with status quo a Walrasian equilibrium* (p^*, x^*) *with* $p^*_N = 1$ *of the economy* $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_M})$, *both* $\widetilde{p}^{*1} \neq 0^{N-1}$ *and* $\widetilde{p}^{*2} \neq 0^{N-1}$.

Therefore, it can be concluded that, generically, both in a political economic equilibrium and a directional political economic equilibrium, Walrasian equilibria are unstable and rationally behaving political candidates have incentives to impose price regulations on the economic system.

5 An Example

In this section the same example as used in Herings (1994) will be analyzed. Consider the political economic system $\overline{\mathcal{E}} = ((X^i, u^i, \omega^i)_{i \in I_2}, (\widetilde{l}, \widetilde{L}), (\widetilde{\mathcal{A}}^k, (\pi^{ik})_{i \in I_2})_{k \in I_2})$ with status quo the Walrasian equilibrium (p^*, x^*) with $p^*_2 = 1$ of the economy $\mathcal{E} = ((X^i, u^i, \omega^i)_{i \in I_2})$, where $N = 2$, $X^1 = X^2 = \mathbb{R}^2_{++}$, $u^1(x^1_1, x^1_2) = (x^1_1)^{\frac{4}{5}} (x^1_2)^{\frac{1}{5}}$, $\forall x^1 \in X^1$, $u^2(x^2_1, x^2_2) = (x^2_1)^{\frac{1}{5}} (x^2_2)^{\frac{4}{5}}$, $\forall x^2 \in X^2$, respectively, $\omega^1 = \omega^2 = (1, 4)^\top$, $(\widetilde{l}, \widetilde{L})$ represents the uniform rationing system, where $\widetilde{l} : Q^2 \to -\mathbb{R}^4_+$ is defined by $\widetilde{l}^1_1(q) = \widetilde{l}^2_1(q) = -2q_1$, $\forall q \in Q^2$, $\widetilde{l}^1_2(q) = \widetilde{l}^2_2(q) = -8q_2$, $\forall q \in Q^2$, and $\widetilde{L} : Q^2 \to \mathbb{R}^4_+$ is defined by $\widetilde{L}^1_1(q) = \widetilde{L}^2_1(q) = 2q_1$, $\forall q \in Q^2$, $\widetilde{L}^1_2(q) = \widetilde{L}^2_2(q) = 8q_2$, $\forall q \in Q^2$,

$$\widetilde{\mathcal{A}}^1 = \widetilde{\mathcal{A}}^2 = \{-1, 0, 1\},$$

and, for every $i \in I_2$, for every $k \in I_2$, $\pi^{ik} : \mathbb{R}_{++} \times \mathbb{R}_{++} \to [0, 1]$ is defined by

$$\pi^{ik}(v^{i1}, v^{i2}) = \frac{\exp(v^{ik})}{\exp(v^{i1}) + \exp(v^{i2})}, \ \forall v^{i1}, v^{i2} \in \mathbb{R}_{++} \times \mathbb{R}_{++}.$$

The unique Walrasian equilibrium (p^*, x^*) with $p^*_2 = 1$ of the economy $\mathcal{E}$ is given by $p^*_1 = 4$, $x^{*1} = (1\frac{3}{5}, 1\frac{3}{5})^\top$, and $x^{*2} = (\frac{2}{5}, 6\frac{2}{5})^\top$, see Herings (1994).

Notice that the Assumptions A1-A4, A6, and A8 are satisfied, except that the range of the utility functions is $\mathbb{R}_{++}$ instead of $(0, 1)$ and hence the domain of the voting functions is given by $\mathbb{R}_{++} \times \mathbb{R}_{++}$ instead of $(0, 1) \times (0, 1)$. Obviously, it is possible to take a monotone transformation of the utility function such that its range becomes $(0,1)$. Notice that also the voting functions have to be transformed in that case. So, a directional political economic equilibrium of the political economic system $\overline{\mathcal{E}}$ with status quo (p^*, x^*) is guaranteed to exist by Theorem 3.9.

A political candidate $k \in I_2$ has three possibilities in this example. Either decrease the price of commodity 1, i.e., $\tilde{p}_1^k = -1$, or stay at the status quo, i.e., $\tilde{p}_1^k = 0$, or increase the price of commodity 1, i.e., $\tilde{p}_1^k = 1$. After some calculations it follows that all possible price regulations $(p, p) \in \underline{P}$ and elements $q \in \tilde{Q}^D(p,p)$ are given by elements of the set S defined by

$$S = \Big\{ (p,p,q) \in \overline{A} \times Q^1 \Big| \begin{array}{l} 0 \leq p_1 \leq 1 \text{ and } q = 1, \text{ or } 1 \leq p_1 \leq 4 \text{ and } q = \frac{2+13p_1}{15p_1}, \text{ or} \\ p_1 = 4 \text{ and } \frac{1}{10} \leq q \leq \frac{9}{10}, \text{ or} \\ 4 \leq p_1 \leq 16 \text{ and } q = \frac{16-p_1}{30p_1}, \text{ or } p_1 \geq 16 \text{ and } q = 0 \Big\}. \end{array} \tag{25}$$

Therefore, it follows that

$$\begin{array}{rcll} \hat{v}^1(p_1) & = & (\frac{-4+9p_1}{5p_1})^{\frac{4}{5}}(\frac{24-4p_1}{5})^{\frac{1}{5}}, & \frac{16}{11} \leq p_1 \leq 4, \\ \hat{v}^1(p_1) & = & (\frac{16+4p_1}{5p_1})^{\frac{4}{5}}(\frac{4+p_1}{5})^{\frac{1}{5}}, & 4 \leq p_1 \leq 16, \end{array}$$

and

$$\begin{array}{rcll} \hat{v}^2(p_1) & = & (\frac{4+p_1}{5p_1})^{\frac{1}{5}}(\frac{16+4p_1}{5})^{\frac{4}{5}}, & 1 \leq p_1 \leq 4, \\ \hat{v}^2(p_1) & = & (\frac{-16+6p_1}{5p_1})^{\frac{1}{5}}(\frac{36-p_1}{5})^{\frac{4}{5}}, & 4 \leq p_1 \leq 11. \end{array}$$

It is easily verified that $\partial_{p_1}\hat{v}^1(4) = -\frac{3}{25}$ and $\partial_{p_1}\hat{v}^2(4) = \frac{3}{25}4^{\frac{3}{5}} \approx 0.276$. Notice that this corresponds to the result of Theorem 3.2, since

$$\begin{array}{rcl} -\partial_{x_2}u^1(x^{*1})(x_1^{*1} - \omega_1^1) & = & -\frac{1}{5}(\frac{1\frac{3}{5}}{1\frac{3}{5}})^{\frac{4}{5}}(\frac{3}{5}) = -\frac{3}{25}, \\ -\partial_{x_2}u^2(x^{*2})(x_1^{*2} - \omega_1^2) & = & -\frac{4}{5}(\frac{\frac{2}{5}}{6\frac{2}{5}})^{\frac{1}{5}}(-\frac{3}{5}) = \frac{3}{25}4^{\frac{3}{5}}. \end{array}$$

It follows that an increase in the price of commodity 1 is harmful to consumer 1, while consumer 2 benefits from such an increase, even when taking into account the resulting supply rationing on the market of commodity 1. This is not surprising since consumer 1 demands commodity 1, while consumer 2 supplies commodity 1. Notice that the benefits for consumer 2 exceed the detrimental effects for consumer 1.

In order to determine the influence of the utility level on the voting behaviour of consumers, $\partial_{v^1}\pi^{i1}(\hat{v}^i(p_1^*), \hat{v}^i(p_1^*))$ and $\partial_{v^2}\pi^{i1}(\hat{v}^i(p_1^*), \hat{v}^i(p_1^*))$ have to be computed for every consumer $i \in I_2$. It follows easily that

$$\begin{array}{rcl} \partial_{v^1}\pi^{i1}\left(\hat{v}^i(p_1^*), \hat{v}^i(p_1^*)\right) & = & \frac{\exp(\hat{v}^i(p_1^*))\exp(\hat{v}^i(p_1^*))}{(\exp(\hat{v}^i(p_1^*))+\exp(\hat{v}^i(p_1^*)))^2} = \frac{1}{4}, \quad \forall i \in I_2, \\ \partial_{v^2}\pi^{i1}\left(\hat{v}^i(p_1^*), \hat{v}^i(p_1^*)\right) & = & \frac{-\exp(\hat{v}^i(p_1^*))\exp(\hat{v}^i(p_1^*))}{(\exp(\hat{v}^i(p_1^*))+\exp(\hat{v}^i(p_1^*)))^2} = -\frac{1}{4}, \quad \forall i \in I_2. \end{array}$$

Moreover,

$$\begin{aligned}
\partial_{v^1}\pi^{i2}\left(\widehat{v}^i(p_1^*),\widehat{v}^i(p_1^*)\right) &= -\partial_{v^1}\pi^{i1}\left(\widehat{v}^i(p_1^*),\widehat{v}^i(p_1^*)\right), \\
\partial_{v^2}\pi^{i2}\left(\widehat{v}^i(p_1^*),\widehat{v}^i(p_1^*)\right) &= -\partial_{v^2}\pi^{i1}\left(\widehat{v}^i(p_1^*),\widehat{v}^i(p_1^*)\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\widetilde{K}^1(\widetilde{p}_1^1,\widetilde{p}_1^2) &= \tfrac{3}{50}(4^{\frac{3}{5}}-1)(\widetilde{p}_1^1-\widetilde{p}_1^2),\ \forall\widetilde{p}_1^1\in\{-1,0,1\},\ \forall\widetilde{p}_1^2\in\{-1,0,1\}, \\
\widetilde{K}^2(\widetilde{p}_1^1,\widetilde{p}_1^2) &= \tfrac{3}{50}(4^{\frac{3}{5}}-1)(\widetilde{p}_1^2-\widetilde{p}_1^1),\ \forall\widetilde{p}_1^1\in\{-1,0,1\},\ \forall\widetilde{p}_1^2\in\{-1,0,1\}.
\end{aligned}$$

The pay-offs of the political candidates of the game $\widetilde{G}=(\widetilde{\mathcal{A}}^1,\widetilde{\mathcal{A}}^2,\widetilde{K}^1,\widetilde{K}^2)$ are given in Figure 5.1. From Figure 5.1 it follows immediately that both political candidates choose

		Political candidate 2		
		-1	0	1
Political candidate 1	-1	$(0,0)$	$(-78,78)$	$(-156,156)$
	0	$(78,-78)$	$(0,0)$	$(-78,78)$
	1	$(156,-156)$	$(78,-78)$	$(0,0)$

Figure 5.1. Pay-offs $\times 1000$ of the political candidates in the example.

to increase the price of commodity 1 in a directional political economic equilibrium of the political economic system $\overline{\mathcal{E}}$ with status quo (p^*,x^*), since $(1,1)$ is a Nash equilibrium in pure strategies of the game $\widetilde{G}$, so the Walrasian equilibrium (p^*,x^*) is unstable as a status quo.

Finally, consider the case where the political candidates have different sets of admissible actions, for example because of commitments made in the past. One political candidate is assumed to have the possibility of lowering the price of commodity 1 compared to the Walrasian equilibrium price, while the other political candidate might propose to increase the price of this commodity. Both political candidates have the possibility to stay at the status quo. Hence,

$$\widetilde{\mathcal{A}}^1=\{-1,0\} \text{ and } \widetilde{\mathcal{A}}^2=\{0,1\}.$$

The pay-offs of the political candidates of the resulting game $\widetilde{G}=(\widetilde{\mathcal{A}}^1,\widetilde{\mathcal{A}}^2,\widetilde{K}^1,\widetilde{K}^2)$ are given in Figure 5.2. From Figure 5.2 it follows immediately that the directional political economic equilibrium of the political economic system $\overline{\mathcal{E}}$ with status quo (p^*,x^*) is given by

		Political candidate 2	
		0	1
Political candidate 1	-1	$(-78, 78)$	$(-156, 156)$
	0	$(0, 0)$	$(-78, 78)$

Figure 5.2. Pay-offs $\times 1000$ of the political candidates in the example.

$(0, 1)$, so political candidate 1 proposes to stay at the status quo, while political candidate 2 proposes to increase the price of commodity 1. Notice that the expected plurality of political candidate 2 is positive in this equilibrium.

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